Risk and Profit in Self-Scheduling for GenCos

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***Abstract*—This letter addresses the risk-based self-scheduling problem of a price-taker Generation Company in the day-ahead competitive electricity markets. The letter analyzes a self-scheduling model that accounts for profit and risk simultaneously. The effect of risk is explicitly modeled in the self-scheduling problem taking into account the variance of the market-clearing prices. The tradeoff of maximum profit versus minimum risk is properly addressed.**

***Index Terms*—Author, please supply your own keywords or send a blank e-mail to keywords@ieee.org to receive a list of suggested keywords.**

NOMENCLATURE

$C_i(\cdot)$	Quadratic cost function of unit i.
$I(i,t)$	The commitment state of unit i at time t.
N	Number of generator units owned by a Generation Company (GenCo).
$N(i,t)$	Non-spinning reserve of unit i at time t.
$P(i,t)$	Generation of unit i at time t.
$R(i,t)$	Spinning reserve of unit i at time t.
r_1, r_2	Probability that spinning and nonspinning reserves are called and generated, respectively.
$S(i,t)$	Startup cost of unit i at time t.
T	Number of hours (24 hours for day-ahead market).
$V_d^{estimated}$	Estimated covariance matrix at day d.
α	Covariance matrix estimation factor.
α^w	Covariance matrix weighting factor.
$\beta(i)$	Risk tolerance parameter of unit i.
$\rho_g^{forecast}(t)$	Forecasted market price for energy at time t.
$\rho_r^{forecast}(t)$	Forecasted market price for spinning reserve at time t.
$\rho_n^{forecast}(t)$	Forecasted market price for nonspinning at time t.
$\boldsymbol{\rho}_w$	Market prices vector at day w.

I. INTRODUCTION

The objective of the Generation Company (GenCo) is to maximize the expected value of profit from selling energy and reserves in the day-ahead competitive electricity markets [1].

In this letter, the self-scheduling problem faced by a GenCo is addressed. The effect of risk is explicitly modeled in the self-scheduling problem taking into account the variance of the market-clearing prices. Therefore, the tradeoff of maximum profit versus minimum risk is properly addressed.

This letter considers the day-ahead energy and reserve markets based on a pool such as New England Power Pool, California Market, New Zealand, and Australia. In these markets, the unit commitment is the responsibility of the individual GenCos in order to maximize their own profit. GenCos' units commitment decision is associated with financial risks. Each supplier would be responsible for its own decision on what and how to bid in different markets.

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II. PROFIT AND RISK MODELING

In competitive electricity markets, GenCos sell power in spot market, sell spinning and nonspinning reserves in reserve markets. Reserve payments can be made in different scenarios [2]. In this letter, reserve is paid only when actually used. The expected value of the profit obtained by a GenCo in the day-ahead electric energy and reserve markets is calculated as

$$\text{Maximize} \sum_{i=1}^{N}\sum_{t=1}^{T} F(i,t) \tag{1}$$

where

$$F(i,t) = \text{Revenue} - \text{Cost}. \tag{2}$$

The GenCo's revenue from the sales of energy and ancillary services (spinning and nonspinning reserves)

$$\begin{aligned}\text{Revenue} = \Big\{&\rho_g^{forecast}(t)P(i,t)I(i,t)\\ &+r_1\rho_r^{forecast}(t)R(i,t)I(i,t)\\ &+r_2\rho_n^{forecast}(t)N(i,t)\Big\}.\end{aligned} \tag{3}$$

The GenCo's production and startup costs are

$$\begin{aligned}\text{Cost} = &\{(1-r_1-r_2)C_i(P(i,t)I(i,t))\}\\ &+\{(r_1-r_2)C_i([P(i,t)+R(i,t)]I(i,t))\}\\ &+\{r_2C_i([P(i,t)+R(i,t)]I(i,t)+N(i,t))\}\\ &+S(i,t)I(i,t)[1-I(i,t-1)].\end{aligned} \tag{4}$$

According to (1), there are T profits calculated at hours $t = 1,2,\ldots,T$ for each unit $i = 1,2,\ldots,N$ owned by a GenCo. These T profits are mutually dependent and this dependence can be measured through their variance which is an appropriate measure of risk. The variance can be computed as

$$\text{Total variance} = \sum_{i=1}^{N} \underset{\rho_1,\ldots,\rho_T}{Var}\left\{\sum_{t=1}^{T} F(i,t)\right\}. \tag{5}$$

The variance of the GenCo's profit equals the variance of revenues minus the variance of the costs. Since the costs are considered deterministic, the variance of profit equals the variance of the revenues as given by (6). Accordingly, (5) can be written using the elements V_{jk} of the $T \times T$ covariance matrix of prices $(\mathbf{V})$ [3] as

$$\begin{aligned}\text{Total variance} = &\sum_{i=1}^{N}\sum_{j=1}^{T}\sum_{k=1}^{T} V_{jk}\\ &\times\{P(i,j)I(i,j)P(i,k)I(i,k)\\ &\quad+R(i,j)I(i,j)R(i,k)I(i,k)\\ &\quad+N(i,j)N(i,k)\}.\end{aligned} \tag{6}$$

The covariance matrix $(\mathbf{V}_d)$ of day d can be estimated using the actual and forecasted values of market prices available up to one day prior to day d

$$\mathbf{V}_d^{estimated} = \frac{1}{N_d}\sum_{w=1}^{N_d}\left(\boldsymbol{\rho}_w^{actual}-\boldsymbol{\rho}_w^{forecast}\right)\left(\boldsymbol{\rho}_w^{actual}-\boldsymbol{\rho}_w^{forecast}\right)^t \tag{7}$$

where N_d is a reasonable number of days (greater or equal to T) for which actual and forecasted market prices are available. This means that the actual and forecasted market prices should be available for at least the last 24 days just prior to the day at which the covariance matrix is to be estimated. Due to the volatile mean and variance of market prices, a modified estimate for the covariance matrix can be obtained using the exponentially weighted moving average equation [4]

$$\mathbf{V}_d^{estimated} = (1-\alpha)\sum_{w=0}^{N_d-1}\alpha^w \times \left(\boldsymbol{\rho}_{N_d-w}^{actual} - \boldsymbol{\rho}_{N_d-w}^{forecast}\right)\left(\boldsymbol{\rho}_{N_d-w}^{actual} - \boldsymbol{\rho}_{N_d-w}^{forecast}\right)^t \quad (8)$$

where α^w is an exponentially decaying weighting factor $(0 < \alpha^w < 1)$, so that higher weights are assigned to days near estimation day d and lower weights are assigned to days a way from the estimation day d. Note that α is the covariance matrix estimation factor which is set to 0.98 in this letter [4].

The GenCos' problem is subject to [1]: System Constraints (GenCos' Energy and Reserve Limits, GenCos' Emission Constraint) and Unit Constraints (Generation Limits, Minimum ON/OFF Durations, Ramping Constraints, Fuel Constraints).

III. Self-Scheduling Model

In this letter, the GenCo is assumed to be price-taker; therefore, the self-scheduling of each generator is independent of the self-scheduling of the others.

A GenCo is interested in maximizing its profit while minimizing risk (variance). In this letter, the technique presented in [5] is used to combine these two conflicting objectives with the help of a risk tolerance parameter $\beta(i)$, $(0 \le \beta(i) < \infty)$, for each unit i owned by the GenCo. The resulting self-scheduling problem has the form

$$\text{Maximize} \quad \sum_{i=1}^{N}\sum_{t=1}^{T} F(i,t) - \sum_{i=1}^{N}\beta(i) \times \left\{\sum_{j=1}^{T}\sum_{k=1}^{T} V_{jk}\{P(i,j)I(i,j)P(i,k)I(i,k) + R(i,j)I(i,j)R(i,k)I(i,k) + N(i,j)N(i,k)\}\right\} \quad (9)$$

which is equivalent to

$$\text{Minimize} \quad \sum_{i=1}^{N}\sum_{t=1}^{T} -F(i,t) + \sum_{i=1}^{N}\beta(i) \times \left\{\sum_{j=1}^{T}\sum_{k=1}^{T} V_{jk}\{P(i,j)I(i,j)P(i,k)I(i,k) + R(i,j)I(i,j)R(i,k)I(i,k) + N(i,j)N(i,k)\}\right\} \quad (10)$$

subject to the system and unit constraints.

The risk tolerance parameter $\beta(i)$ represents the tradeoff between GenCo's expected profit and risk. A conservative GenCo chooses large values for $\beta(i)$ to increase the weight of the risk measure in (10). However, a GenCo looking for higher profit chooses low values for $\beta(i)$.

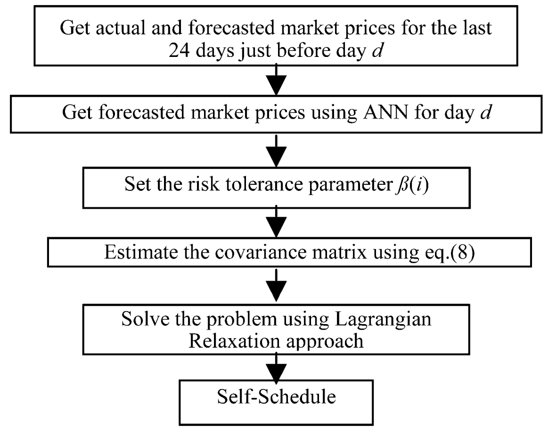

Fig. 1. Flowchart of proposed self-scheduling model at day d.

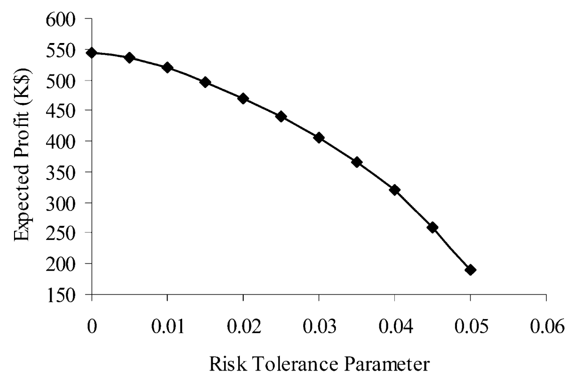

Fig. 2. GenCo's expected profit versus risk tolerance parameter.

IV. Solution Methodology

The self-scheduling model is formulated as a mixed-integer quadratic programming problem, which is solved using Lagrangian Relaxation. We write the Lagrangian function, omitting constant terms, by adding the relax system constraints using Lagrangian multipliers to the objective function (10). The convergence criterion may be defined as the relative duality gap to be less than a pre-specified threshold [1]. The flowchart of the proposed self-scheduling model is shown in Fig. 1.

Case Study: We use a GENCO with 36 generating units to illustrate the proposed model. Unit data, forecasted demand and forecasted market prices are given in [1], [6]. The spinning and nonspinning reserves are set to 5% and 3% of the forecasted system demand respectively. The system peak demand is 4242.0 MW at hour 18. A study period of 24 hours of winter weekends, weeks 44–52, is considered [1]. The probability that spinning and nonspinning reserves are called and generated is set to 0.005 and 0.003, respectively.

To forecast prices, a data of nine months corresponding to the electric energy and reserve markets of California [6] have been used. Price forecasts are obtained using an ANN approach [1]. The actual and forecasted market prices for the last 24 days just prior to the day of estimation d are used to estimate the covariance matrix at day d by using (8). The self-scheduling problem given by (10) is solved for different values of the risk parameter $\beta(i)$, which allows assigning different weights to the risk term versus the profit term in the objective function. In this letter, for simplicity, the risk tolerance parameter is assumed to be the same for all units owned by a GenCo.

Fig. 2 shows the GenCo's expected profit at different risk levels. It is clear that the expected profit is crucially affected by the level of risk that the GenCo is willing to take. If the probability that reserves are

TABLE I
GENCO'S EXPECTED PROFIT AND PROBABILITY THAT RESERVES ARE CALLED AND GENERATED ($\beta(i) = 0$)

Probability		GenCo's Expected Profit ($)
r_1	r_2	
0.005	0.003	545,220.5
0.025	0.015	551,435.0
0.05	0.03	564,178.4

called and generated is increased then the GenCo's profit increases as shown in Table I. Large values of r_1 and r_2 means that the reserves have more chance to be called. The results in Table I show how the profit is sensitive to r_1 and r_2.

V. CONCLUSIONS

In the competitive electricity markets, GenCos focus on the tradeoff between maximum profit and minimum risk. This letter presents a model for analyzing the tradeoff between profit and risk faced by GenCos in the day-ahead competitive energy and reserves markets. This model is formulated as a mixed-integer quadratic programming problem, which is solved using Lagrangian Relaxation approach. Simulation results using a 36-unit GenCo show that profit is crucially affected by the desired level of risk.

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Pop-Up Generator Step-Up: A Narrow Look at the State of U.S. and Canadian Transmission Model Data

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***Abstract*—A source of error in many large-scale power flow and transient stability models are the "pop-up" Generator Step-Up models. These models allow the electrical system planner to create equivalent high-side generator models for power flow simulations, yet allows the generator low side to be modeled for transient stability simulations. The concept was originally introduced to reduce the number of power flow buses needed to model the electrical system during the early years when computer memory was restricted. If perfectly applied, there is nothing wrong with this concept. However, in real world applications, it fails to provide feedback to the planner that an improper model has been prepared. This paper describes misapplied differences that introduce errors. Probably the most significant problem is that "sanity checking" of the resultant model cannot be easily performed. The review of this narrow issue provides significant insight into the state of U.S. and Canadian models for power flow and transient stability. The authors recommend that the use of pop-up models be eliminated in modeling environments such as the NERC MMWG. We further conclude that additional engineering checking of the data would be productive.**

I. INTRODUCTION

In recent transient stability studies, we have noticed that a significant portion of the large dynamic models contain data that is almost certainly in error. A major contributor to this problem appears to be the pop-up Generator Step-Up (GSU) transformer models originally available in some power system analysis programs. The pop-up GSU model allows the electrical system planner to create equivalent generator models on the high-side GSU bus for power flow models, yet model the generator on its low side for transient stability simulations. The concept of the pop-up generator was originally introduced to reduce the number of power flow buses needed to model the electrical system for both power flow and transient stability simulations during the early years when computer memory was restricted. From roughly 1960 through the mid 1980s, available memory (and processor speed) generally restricted major power flow programs to between 1000 and 10 000 buses. The use of the pop-up generator model allowed the generator bus to be implicitly modeled for transient stability where it was needed and to be essentially ignored in power flow where, if proper care was taken, it was not needed for good simulations. This removed several hundred buses from these early models.

II. INCREASING CAPABILITIES

In the last 15 years or so, Moore's law has resulted in a tremendous boost in available memory capacity and processor speed. Modern power flow and transient stability programs can now routinely handle up to 50 000 buses. Larger power system models (such as the MMWG/eastern interconnection model, circa 2003 [3]) are approaching 40 000 buses in size. In fact, the current capability limits of modern programs have more to do with the size of available models than with the capability of the computers.

As a result of the increased capability to handle large models, it is now recommended that users not use pop-up models. Instead, the user should explicitly model the generator on its low-side bus and include

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