Short-Term Transmission Line Maintenance Scheduling in a Deregulated System

M. K. C. Marwali and S. M. Shahidehpour

Abstract—**In this paper we present a decomposition approach based on duality theory for line maintenance scheduling with transmission and voltage constraints. The given formulation consists of a master program and sub-problems with two independent programming. In the master problem, maintenance problem is solved and in the subproblems, transmission and voltage problems are solved independently. Since canceling a transaction or purchasing reactive power relates to the loss of revenue, the trade-off between maintenance cost and revenue loss will be optimized in the proposed method. The test results on the modified IEEE 118-bus system demonstrate that limits on transmission and voltage affect the line maintenance scheduling and increase the maintenance cost. While introducing the loss of revenue as one of the objective functions, the proposed method is flexible enough to accommodate various pricing objectives and methods.**

Index Terms—**Decomposition, short-term scheduling, transmission maintenance, transmission management, transmission network.**

I. INTRODUCTION

THE RAPIDLY changing business environment for electric power utilities has resulted in unbundling of services provided by these utilities. Wheeling of electrical energy (transmission services) is one of the more prevalent of such unbundled services. The changing structure of electric power utilities and the emergence of independent power producers, independent customers and transmission provider are bringing transmission management into a new focus.

The United States Federal Energy Regulatory Commission (FERC) addresses some of the complexities surrounding transmission issues. The FERC paper [6] deals with political and economic issues in details but does not cover engineering issues. Operation, maintenance and expansion of transmission systems that concern transmission services are addressed in [1]–[5]. Papers [2], [3], [5] address rate design for wheeling services. In fact, evaluating transmission costs remain to be a difficult task that requires complex analytical tools and extensive data.

An integrated planning decision that confronts transmission providers is the line maintenance scheduling as it may entail to a possibility of revenue loss. Suppose that a transmission provider must consider line maintenance in a particular week and has the option of choosing the appropriate time during that week. It may be worth scheduling the line maintenance during low wheeling hours of the day. However, since low wheeling period may not occur during regular working hours, it may require paying overtime to the maintenance crew. There is usually a trade-off in choosing the best time for line maintenance during the 24-hour period.

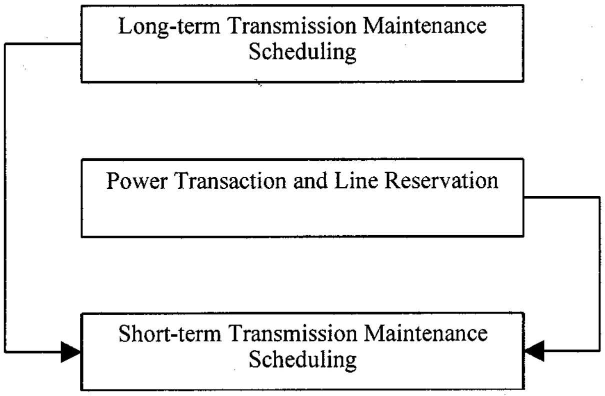

Fig. 1. Proposed flowchart.

As the maintenance schedule of a transmission line spans over different time periods, its impact on the reliability of the overall system must be considered. In this connection, following functions (Fig. 1) are identified:

1) Long-term Line Maintenance Scheduling (LTS): The long-term maintenance period of one year is divided into intervals (weeks) and the "best" maintenance scheduling strategy is derived to satisfy weekly line maintenance constraints, seasonal constraints, and the system energy expressed as load duration curves in each interval. To maintain system reliability. Details of LTS can be found in [14].
2) Power Transaction and Line Reservation: Power transactions and schedule for supplying loads are determined in this step. The schedule may arrive after generation providers have been able to sell power to retailers, wholesalers or direct-access customers through bilateral contracts, as well as sale to the Power Exchange. Transmission contracts are based on usage, and path services are based on reservations [5]–[7].
3) Short-term Line Maintenance Scheduling: Given the maintenance window from LTS, and transaction schedules from Step 2), the short-term line maintenance is then formulated to minimize the transmission provider's loss of revenue while satisfying hourly line maintenance constraints, line reservations and system reliability.

Manuscript received February 14, 2000. This work was supported by a Grant from the Siemens Corporation in Brooklyn Park, MN.

The authors are with the Department of Electrical and Computer Engineering, Illinois Institute of Technology, Chicago, IL 60616 (e-mail: ms@iit.edu).

Publisher Item Identifier S 0885-8950(00)08278-X.

In this paper, we propose a solution to the problem of short-term transmission maintenance to maximize transmission provider's revenue while satisfying system constraints. This paper is not meant to debate advantages or disadvantages of various pricing policies, such as embedded cost pricing, marginal cost pricing or value-based pricing. We also stay clear of all policy issues related to transmission access. Our emphasis on transmission maintenance throughout this paper is to illustrate how to take into account cost-tradeoffs and constraints that are involve in evaluating the impact of line maintenance schedule. Though introducing loss of revenue as the objective function, the proposed method is flexible enough to accommodate various pricing methods.

II. PROBLEM FORMULATION

The difference between short-term and long-term maintenance scheduling is the time horizon and the time increment. In LTS, the horizon of study is one or two years with increment of a week (Δt), but in short-term maintenance scheduling the horizon of study is weeks with increment of an hour $(\Delta \tau)$. The window for short-term line maintenance scheduling is determined by LTS, hence short-term maintenance scheduling only needs to determine specific maintenance hours within that window. List of symbols is given in Appendix A.

The objective of short-term transmission maintenance scheduling is to minimize maintenance cost and loss of revenue due to line maintenance while satisfying maintenance requirements,

$$\begin{aligned}\text{Min} \sum_{\tau} \sum_{k\in\omega} &\{C'_{k\tau}(\overline{N}_k - N_{k\tau})\} \\ &+ \sum_{\tau} \sum_{i\in\Omega} \{c_{i\tau} r_{i\tau}\} + \sum_{\tau} \sum_{i\in\beta} \{c'_{i\tau} s_{i\tau}\} \end{aligned} \tag{1}$$

The first term of objective (1) is the transmission line maintenance cost. The set of lines that require maintenance is ω. The second term is the loss of revenue due to line maintenance. The loss of revenue is proportional to the amount of scheduled energy that cannot be delivered to receiving buses due to line maintenance. This implies that a few contracts that are recallable may ought to be canceled. The set of receiving buses where there are recallable active power contracts is Ω.

Since the objective is to maximize transmission provider's revenues, the cheapest maintenance schedule may interrupt the flow of real or reactive power on certain lines and cause violation of line flows or bus voltages. A transmission provider has two options to provide additional reactive power. The first option is to interrupt its reactive power contracts. The second option is to purchase reactive power from ancillary service providers. The cost of these two options is represented by the third term of objective (1). The set of buses where there are additional reactive power or recallable reactive power contracts available is β.

We introduce variable y as a decision variable. Its value is 0 if M_k lines in the right-of-way k are off-line for maintenance otherwise its value is 1.

$$\begin{aligned} N_{k\tau} &= (\overline{N}_k - M_k) + y_{k\tau} M_k \\ y_{k\tau} &\in \{0, 1\} \end{aligned} \tag{2}$$

then (1) becomes

$$\begin{aligned}\text{Min} \sum_{\tau} \sum_{k\in\omega} &\{C'_{k\tau} M_k (1 - y_{k\tau})\} \\ &+ \sum_{\tau} \sum_{i\in\Omega} \{c_{i\tau} r_{i\tau}\} + \sum_{\tau} \sum_{i\in\beta} \{c'_{i\tau} s_{i\tau}\} \end{aligned} \tag{3}$$

The sets of constraints are given as follows:

1) Maintenance Constraints: Constraints (4) represent the maintenance window stated in terms of maintenance variable $y_{k\tau}$. Lines must be available before their earliest possible period maintenance (e'_k) and after their latest possible period of maintenance (e.g., $l'_k + d'_k$. The values of e'_k and $l'_k + d'_k$ have been determined by LTS. Hence the study horizon of short-term line maintenance scheduling is between e'_k and $l'_k + d'_k$.

$$\begin{aligned} &y_{k\tau} = 1 && \text{for } \tau \le e'_k \text{ or } \tau \ge l'_k + d'_k \\ &\left(Y^{\text{off}}_{k(\tau-1)} - d'_k\right) \\ &\quad \cdot (y_{k\tau} - y_{k(\tau-1)}) \ge 0 && \text{for } e'_k \le \tau \le l'_k \\ &\sum_{\tau} y_{kt} = l'_k - e'_k - d'_k && \text{for } e'_k \le \tau \le l'_k \end{aligned} \tag{4}$$

Additional constraints consisting of crew and resource availability, seasonal limitations, and pre-scheduling can be incorporated into e'_k and l'_k of constraint (4). If for example we consider that lines 1, 2 and 3 should be simultaneously on maintenance, the set of constraints is formulated as follows:

$$y_{1\tau} + y_{2\tau} + y_{3\tau} = 3 \quad \text{or} \quad y_{1\tau} + y_{2\tau} + y_{3\tau} = 0$$

If we consider that in each maintenance area we have limited resources and crew, a new set of constraints is formulated as follows:

$$\sum_{k\in A} \sigma_{mk} M_k (1 - y_{k\tau}) \le \psi_{m\tau}$$

In the case of resource constraints, $\psi_{m\tau}$ would be the amount of resource m available in area A for each hour τ and σ_{mk} would be a percentage of this resource required for the maintenance of line k. In the case of crew constraint, $\psi_{m\tau}$ would be the number of maintenance crews of type m in area A and σ_m would be a percentage of maintenance crew required for maintenance of line k.

2) Network Constraints: Network constraints are decomposed into two smaller problems. The first is a set of constraints on real power flow which limits the violations of transmission security constraints based on the worst contingency case. The second is a set of constraints on reactive power which examines voltage constraints. The two sets of constraints are represented as follows.

- *Limits on Real Power Flows:* Since we are primarily interested in screening of transmission violations, we apply a dc load flow to expedite the screening process. In this model, active power flows obey the two Kirchoff laws (node law and loop law). The active power flow between buses i and j is calculated as:

$$f_k = (\theta_i - \theta_j)\gamma_k \tag{5}$$

The constraints corresponding to flow limits are:

$$|f_k| \le \overline{f_k} \tag{6}$$

The flow limits are written in terms of bus voltage angle θ by substituting (5) into (6):

$$|\theta_i - \theta_j| \le \overline{\psi_k} \tag{7}$$

where $\overline{\psi}_k = \overline{f}_k/\gamma_k$ is the maximum angle across branch k.

Assuming generation schedule is known from power transactions, network constraints are expressed in a matrix as follows.

$$\begin{aligned} \boldsymbol{B}\theta &= \boldsymbol{d} - \boldsymbol{g} \\ |\boldsymbol{S}'\theta| &\le \overline{\psi} \end{aligned} \tag{8}$$

Let $\boldsymbol{P} = \boldsymbol{d} - \boldsymbol{g}$, then (8) become:

$$\begin{aligned} \boldsymbol{B}\theta &= \boldsymbol{P} \\ |\boldsymbol{S}'\theta| &\le \overline{\psi} \end{aligned} \tag{9}$$

Constraints (9) represent load balance and other operational constraints such as transmission capacity limits.

- *Limits on Voltages:* Transmission providers have to be responsible for maintaining the quality of service that may have declined because of line maintenance. This goal can be accomplished by canceling reactive power sale transactions or purchasing additional reactive power, adjusting tap-changing transformers. The reactive power constraints are:

— Reactive power operating reserve requirement

$$\sum_i \overline{Q}_i \ge Q_D \tag{10}$$

— Reactive power generation limits and load bus balance

$$\begin{aligned} \underline{\boldsymbol{Q}}_G &\le \boldsymbol{Q}_G = \boldsymbol{F}_1(V) \le \overline{\boldsymbol{Q}}_G \\ \boldsymbol{Q}_L &= \boldsymbol{F}_2(\boldsymbol{V}) \end{aligned} \tag{11}$$

— System voltage and transformer tap limits

$$\begin{aligned} \underline{\boldsymbol{V}} &\le \boldsymbol{V} \le \overline{\boldsymbol{V}} \\ \underline{\boldsymbol{T}} &\le \boldsymbol{T} \le \overline{\boldsymbol{T}} \end{aligned} \tag{12}$$

We assume $\boldsymbol{V}^0$ is the system voltage vector based on initial unit commitment state. If $\boldsymbol{V}^0$ is infeasible, additional adjustments to the generation and/or distribution of reactive power may be necessary in order to shift the system voltage from $\boldsymbol{V}^0$ to a desired value $\boldsymbol{V}^*$. We neglect the effect of reactive power adjustment on bus voltage angles. Equation (11) is linearized in the vicinity of the initial operating point $\boldsymbol{V}^0$ and the effect of tap-changing transformer on voltages is considered in the linearized form [15] as follows:

$$\begin{aligned} Q_G^0 + J_1''\Delta V &\le \overline{Q}_G + \Delta\overline{Q}_T \\ J_2''\Delta V &= 0 \\ \Delta\underline{\boldsymbol{V}} &\le \Delta\boldsymbol{V} \le \Delta\overline{\boldsymbol{V}} \end{aligned} \tag{13}$$

where $\Delta\boldsymbol{V} = \boldsymbol{V}'' - \boldsymbol{V}^0$ is the incremental system voltage for $\Delta\boldsymbol{Q}_G$. $\boldsymbol{J}_1''$ is the modified Jacobian matrix for buses connected to generators and transformers, and $\boldsymbol{J}_2''$ is for load buses.

3) Line Reservation and Ancillary Services: The transmission provider may offer firm and nonfirm transmission reservations. In firm reservations, curtailment does not apply to situations in which transmission is discontinued for economic reasons. Transmission service is to be curtailed only in cases where system reliability is threatened or emergency conditions exist. Firm transmission service is mostly provided to utilities native loads and, under contract, to firm wheeling transactions. In nonfirm reservations, transmission provider has the right to interrupt all or part of transmission services for any reasons, including economic.

To accommodate firm and nonfirm reservations in our formulation, we include them in flow constraints and the objective function. The firm reservation on line k at hour τ can be represented as minimum flow constraint $\underline{f_{k\tau}}$. The operation model corresponding to flow limits becomes:

$$\underline{f_{k\tau}} \le |f_{k\tau}| \le \overline{f_k} \tag{14}$$

To accommodate the possibility for canceling nonfirm reservations, we introduce a slack variable $\boldsymbol{r}$ (load curtailment) into (9). The load curtailment, $r_{i\tau}$, should be smaller than or equal to the recallable load at bus i and hour τ. The active power constraints become:

$$\begin{aligned} \boldsymbol{B}\theta + \boldsymbol{r} &= \boldsymbol{P} \\ \boldsymbol{r} &\le \overline{\boldsymbol{r}} \\ |\boldsymbol{S}'\theta| &\le \overline{\psi} \end{aligned} \tag{15}$$

Slack variable $r_{i\tau}$ exists only at buses where there are recallable transactions. Additional reactive power may be required to support the voltage profile. We introduce slack variable $\boldsymbol{s}$ into (13) as additional injected reactive power and let $\overline{Q} = \overline{Q}_G + \Delta\overline{Q}_T + Q_G^0$.

$$\begin{aligned} J_1''\Delta V - s &\le \overline{Q} \\ J_2''\Delta V &= 0 \\ \Delta\underline{\boldsymbol{V}} &\le \Delta\boldsymbol{V} \le \Delta\overline{\boldsymbol{V}} \\ s &\le \overline{s} \end{aligned} \tag{16}$$

Since canceling a transaction or purchasing reactive power relates to the loss of revenue, the trade-off between maintenance cost and revenue loss will be optimized in the proposed method. A transmission provider has two options to provide additional reactive power due to line maintenance. The first option is to interrupt recallable loads. The second option is to purchase reactive power from ancillary services. Slack variables $s_{i\tau}$ exist only at buses where there are recallable loads or ancillary services.

III. SOLUTION METHODOLOGY

The proposed method is depicted in Fig. 2, which uses a Benders decomposition. The master problem solves the short-term maintenance and the subproblem checks the operational (i.e., line flow and voltage) constraints for the proposed maintenance solution. If violations are observed in the subproblem, Benders cuts will be set up and added to the master problem to generate a revised solution. The detailed discussion for each step is given next.

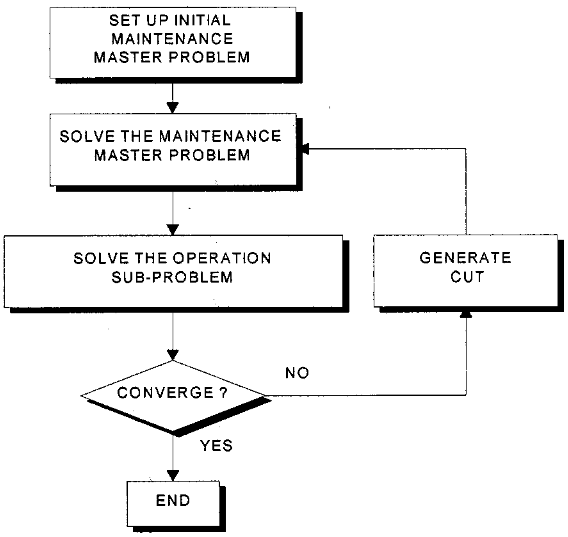

Fig. 2. Solution methodology.

A. Initial Master Problem

The initial maintenance master program is formulated as follow:

$$
\begin{aligned}
\text{Min}\quad & z \\
\text{S.T.}\quad & z \geq \sum_{t} \left\{ \sum_{k\in\omega} C'_{kt} M_k (1 - y_{k\tau}) \right\} \\
& y_{k\tau} = 1 && \text{for } \tau \leq e'_k \\
& && \quad \text{or } \tau \geq l'_k + d'_k \\
& \left(Y^{\text{off}}_{(\tau-1)} - d'_k\right) \\
& \quad \cdot \left(y_{k\tau} - y_{k(\tau-1)}\right) \geq 0 && \text{for } e'_k \leq \tau \leq l'_k \\
& \sum_{\tau} y_{kt} = l'_k - e'_k - d'_k && \text{for } e'_k \leq \tau \leq l'_k \\
& \sum_{k\in A} \sigma_{mk} M_k (1 - y_{k\tau}) \leq \psi_{m\tau}
\end{aligned}
\tag{17}
$$

Optimization (17) may be seen as a discrete optimization with decision variable y_{kt}. Branch and bound, dynamic programming or any other integer programming can solve this optimization. We use the branch and bound method to solve (17).

B. Operation Sub-Problems

Network constraints are independent of time period and are categorized as decoupling constraints; those constraints that are dependent on the time period are categorized as coupling constraints. Maintenance constraints (4) may be considered as coupling constraints. The decoupling structure of network constraints makes it possible to solve them independently in each time period. Transmission line revenues depend on the availability of lines for wheeling power in each time period, hence, loss of revenues if one or more transmission lines are on maintenance, w_τ, may be expressed as:

$$
\begin{aligned}
\text{Min}\quad & w_\tau = \sum_{i\in\Omega} \{c_{i\tau} \boldsymbol{r}_{i\tau}\} + \sum_{i\in\beta} \{c'_{i\tau} s_{i\tau}\} \\
\text{S.T.}\quad & \boldsymbol{B}\theta + \boldsymbol{r} = \boldsymbol{P} \\
& \boldsymbol{r} \leq \overline{\boldsymbol{r}} \\
& \underline{\psi} \leq |\boldsymbol{S}'\theta| \leq \overline{\psi} \\
& \boldsymbol{J}''_1 \Delta \boldsymbol{V} - \boldsymbol{s} \leq \overline{\boldsymbol{Q}} \\
& \boldsymbol{J}''_2 \Delta \boldsymbol{V} = 0 \\
& \Delta \underline{\boldsymbol{V}} \leq \Delta \boldsymbol{V} \leq \Delta \overline{\boldsymbol{V}} \\
& \boldsymbol{s} \leq \boldsymbol{s}
\end{aligned}
\tag{18}
$$

The formulation (18) consists of two independent sets which are decomposed into two sub-problems as follows.

- *Transmission Limits Sub-Problem:* Maintenance decision $\boldsymbol{y}$ in the active power flow model also corresponds to line capacities. However, a transmission line in this model is characterized by two parameters: flow capacity $\overline{f}$ and susceptance γ. Observing (18), we notice that γ is not in the righthand side of the problem but is part of the $\boldsymbol{B}$ matrix. Susceptance γ can be written explicitly as a decision variable if we rewrite transmission limit sub-problem in (19). Here, we define $w1^n_\tau$ as the loss of revenue for hour τ associated with the nth trial solution. Multiplier λ^n may be interpreted as the marginal loss of revenue associated with 1 MW decrease in the line capacity, given the nth trial maintenance schedule. Multiplier ξ^n may be interpreted as the marginal loss of revenue associated with 1/Ohm decrease in the line susceptance, given the nth trial maintenance schedule.

$$
\begin{aligned}
\text{Min}\quad & w1_\tau = \sum_{i\in\Omega} c_{i\tau} \boldsymbol{r}_{i\tau} \\
\text{S.T.}\quad & \boldsymbol{S}\boldsymbol{f} + \boldsymbol{r} = \boldsymbol{P} && \text{(dual variable is } \upsilon) \\
& \boldsymbol{r} \leq \overline{\boldsymbol{r}} \\
& \underline{\boldsymbol{f}} \leq \boldsymbol{f} \leq \overline{\boldsymbol{f}}(\overline{\boldsymbol{N}} - \boldsymbol{M}(1 - \boldsymbol{y}^n)) && \text{(dual variable is } \lambda) \\
& \boldsymbol{f}/\psi = \gamma(\overline{\boldsymbol{N}} - \boldsymbol{M}(1 - \boldsymbol{y}^n)) && \text{(dual variable is } \xi)
\end{aligned}
\tag{19}
$$

The sub-problem (19) is nonlinear due to the ratio of f/ψ. It is shown in [11] that it will be not necessary to solve the nonlinear problem as formulated in (19). We solve active power part of the linear problem (18) instead of the nonlinear problem (19).

$$
\begin{aligned}
\text{Min}\quad & w1_\tau = \sum_{i\in\Omega} \{c_{i\tau} \boldsymbol{r}_{i\tau}\} \\
\text{S.T.}\quad & \boldsymbol{B}\theta + \boldsymbol{r} = \boldsymbol{P} \\
& \boldsymbol{r} \leq \overline{\boldsymbol{r}} \\
& \underline{\psi} \leq |\boldsymbol{S}'\theta| \leq \overline{\psi}
\end{aligned}
\tag{20}
$$

The multiplier ξ^n_k associated with the sensitivity of the susceptance of branch k (between buses i and j) is calculated as:

$$
\xi^n_k(i, j) = (\upsilon^n_i - \upsilon^n_j)(\theta^n_j - \theta^n_i) \tag{21}
$$

- *Voltage Limits Sub-Problem:* The maintenance decision $\boldsymbol{y}$ in reactive power flow model corresponds to $\boldsymbol{J}''$ matrix. Observing (18), we notice that $\boldsymbol{J}''$ is not in the right-hand side of the problem. We apply a procedure similar to the active power flow problem. First, we solve the reactive power part of the linear problem (18).

$$\begin{aligned} \text{Min} \quad & w2_\tau = \sum_{i\in\beta} c'_{i\tau} s_{i\tau} \\ \text{S.T.} \quad & J_1'' \Delta V - s \le \overline{Q} \qquad (\text{dual varible is } \sigma) \\ & \boldsymbol{J_2''} \Delta \boldsymbol{V} = 0 \\ & \Delta \underline{\boldsymbol{V}} \le \Delta \boldsymbol{V} \le \Delta \overline{\boldsymbol{V}} \\ & \boldsymbol{s} \le \overline{\boldsymbol{s}} \end{aligned} \tag{22}$$

We define multiplier μ_k as the sensitivity of J_k'' in which J_k'' is the element (i, j) of matrix J_1''. Then multiplier μ_k^n is calculated from the multiplier of (22) as:

$$\mu_k^n(i, j) = (\sigma_i^n - \sigma_j^n)(V_j^n - V_i^n) \tag{23}$$

Given the dual multipliers in (21) and (23), the associated Benders cut of (18) is:

$$\begin{aligned} z \ge & \sum_\tau \sum_k \{C'_{k\tau} M_k (1 - y_{k\tau}) \\ & \qquad -c_{k\tau} f_{k\tau} \left(\overline{N}_{k\tau} - M_k (1 - y_{k\tau})\right)\} \\ & + \sum_\tau w1_\tau^n + \sum_\tau w2_\tau^n + \sum_\tau \sum_k \\ & \cdot \{\lambda_{k\tau}^n \overline{f} M_k + \xi_{k\tau}^n \gamma_k + \mu_{k\tau}^n J_k''\} (y_{k\tau}^n - y_{k\tau}) \end{aligned} \tag{24}$$

where n is the index of iteration and λ^n, ξ^n, μ^n are multipliers in the nth iteration. The cut (24) will tend to increase lower bounds obtained from successive maintenance sub-problem solutions.

C. Revised Master Problem

Operation sub-problems yield a set of dual multipliers from which a cut is constructed in each iteration. This cut is added to masterproblem to revise the previous master-problem. The master-problem at iteration n is

$$\begin{aligned} \text{Min} \quad & z \\ \text{S.T.} \quad & z \ge \sum_t \left\{ \sum_{k\in\omega} C'_{kt} M_k (1 - y_{k\tau}) \right\} \\ & y_{k\tau} = 1 \qquad \text{for } \tau \le e'_k \\ & \qquad\qquad \text{or } \tau \ge l'_k + d'_k \\ & \left(Y_{(\tau-1)}^{\text{off}} - d'_k\right) \\ & \quad \cdot \left(y_{k\tau} - y_{k(\tau-1)}\right) \ge 0 \qquad \text{for } e'_k \le \tau \le l'_k \\ & \sum_\tau y_{kt} = l'_k - e'_k - d'_k \qquad \text{for } e'_k \le \tau \le l'_k \\ & \sum_{k\in A} \sigma_{mk} M_k (1 - y_{k\tau}) \le \psi_{m\tau} \\ & z \ge \sum_\tau \sum_k \{C'_{k\tau} M_k (1 - y_{k\tau}) \\ & \qquad -c_{k\tau} f_{k\tau} \left(\overline{N}_{k\tau} - M_k (1 - y_{k\tau})\right)\} \\ & \quad + \sum_\tau w1_\tau^n + \sum_\tau w2_\tau^n + \sum_\tau \sum_k \\ & \quad \cdot \{\lambda_{k\tau}^n \overline{f} M_k + \xi_{k\tau}^n \gamma_k + \mu_{k\tau}^n J_k''\} (y_{k\tau}^n - y_{k\tau}) \\ & y_{k\tau} \in \{0, 1\} \end{aligned} \tag{25}$$

The important feature of the Benders decomposition is the availability of upper and lower bounds to the optimal solution at each iteration. These bounds are used as an effective convergence criterion. The convergence criterion in Fig. 2 is:

$$\frac{2(z^n - z^{n-1})}{(z^n + z^{n-1})} \le \Delta_n \tag{26}$$

where z^n is the value of z at iteration n in the loop. There is a trade-off between Δ_n and the number of iterations. The smaller the Δ_n the larger the number of iterations. For some cases, the problem may not converge if the Δ_n is too small. From our experience, setting Δ_n to 1% is good enough.

IV. TEST CASES

We use the 186-line IEEE-118 bus network to test the proposed method. Transmission lines have a 300 MW capacity and a step-size for reactive problem is used as $\Delta V_{step} = 0.25$, $\Delta Q_{step} = 0.1$. In our example, the program is executed for a 24-hour period to analyze the results, however, the program can easily be applied to a week period for practical purposes. Hourly total transactions are given in Table B1. Table B2 gives the data for lines that require maintenance. The maintenance duration of each line is one hour. The maintenance cost for the 10 lines considered in this example is given in Table B3. Transmission providers may relieve a contingency in the system by interrupting recallable contracts. The corresponding loss of revenue is given in Table B4. Based on the static security analysis, the worst transmission contingency is chosen as the outage of line 37–38 for security assessment.

The transmission provider has two options to provide additional reactive power which may be required due to line maintenance. The first option is to interrupt reactive power delivery. The loss of revenue of this action is given in Table B4. The second option is to purchase reactive power from ancillary services. The cost of purchasing additional reactive power is given in Table B5.

The minimization of maintenance costs and the loss of revenue are used as the objective function. The results of the following test cases are included to show the effect of transmission and voltage constraints on the line maintenance scheduling.

- Case 0: We consider only line maintenance constraints with no network constraints
- Case 1: We consider line maintenance and transmission constraints but no voltage constraints
- Case 2: In addition to Case 1, voltage constrains are imposed on line maintenance

Case 0 is a classical integer programming optimization since there are no transmission and voltage constraints. In Case 0, no contract has been canceled. The final maintenance cost is given in Table I. The corresponding hourly line maintenance schedule is shown in Table II. The numbers (1 or 0) in Table II represent on/off states of lines for maintenance at different hours: off indicates that the line is unavailable and scheduled for maintenance. It is obvious that the schedule gives the minimum maintenance cost since most lines are scheduled between hours 9 and 17; Table B3 indicates that during these hours the maintenance cost is the cheapest. Now we check the operation constraints.

TABLE I
LINE MAINTENANCE COST

Case	Cost ($)
0	2292.84
1	4655.64
2	8244.08

TABLE II
LINE MAINTENANCE SCHEDULE WITHOUT NETWORK CONSTRAINTS (CASE 0)

Line	Hour (1-24)																							
	1					6						12						18						24
1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1

TABLE III
OVERFLOW ON TRANSMISSION LINES (CASE 0)

Line Between Buses	MW Rating	Peak MW Flow
8-30	300	302.56
69-3	300	300.12
89-92	300	421.98

TABLE IV
BUS VOLTAGES WITHOUT CONSTRAINTS IN LOCAL AREA (CASE 0)

Bus No	Voltage (p.u.)					
	Hour 1	Hour 2	Hour 14	Hour 15	Hour 16	Hour 24
104	0.9143	0.9203	0.9177	0.9187	0.9187	0.9475
105	0.9096	0.9165	0.9156	0.9167	0.9167	0.9397
106	0.9014	0.9092	0.9048	0.9060	0.9060	0.9426
107	**0.8854**	**0.8955**	**0.8863**	**0.8879**	**0.8879**	0.9215
108	0.9091	0.9158	0.9336	0.9346	0.9346	0.9295
109	0.9096	0.9161	0.9412	0.9422	0.9422	0.9260
110	0.9159	0.9212	0.9658	0.9666	0.9666	0.9216
112	**0.8882**	**0.8961**	0.9361	0.9373	0.9373	**0.8929**

The maximum transmission flow over the 24 hour study period is shown in Table III. In this case, there are flow violations on lines 8–30, 69–3 and 89–92. There are also voltage violations at buses 107 and 112 as seen in Table IV; the violated voltages are shown as bold. Most of these violations occur during heavy transaction periods.

We now solve Case 1 in our study. The imposed transmission constraints increase the cost of maintenance in Case 1. To satisfy transmission constraints, 110.6 MW of recallable contracts at bus 42 have been interrupted at hours 19 and 21. Table I shows a change in operating cost over the study period, indicating a shift in the maintenance schedule. The maintenance schedule which satisfies transmission security constraints, is given in Table V. There are no transmission lines on maintenance during hours 9–14 due to heavy transactions in these hours. Lines 2, 3, 7, 8, 9 and 10 are forced to be on maintenance with additional costs to avoid flow violations in the network (see Table B3). Transmission constraints reduce voltage violations but do not guarantee that all bus voltages will be within limits. The bus voltages after transmission constraints are imposed can be seen in Table VI. Voltages in a local area are low, as the local area does not have enough reactive power generation.

TABLE V
LINE MAINTENANCE SCHEDULE WITH TRANSMISSION CONSTRAINTS (CASE 1)

Line	Hour (1-24)																							
	1					6						12						18						24
1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
7	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1

TABLE VI
BUS VOLTAGES WITHOUT VOLTAGE CONSTRAINTS IN LOCAL AREA (CASE 1)

Bus No	Voltage (p.u.)					
	Hour 1	Hour 2	Hour 14	Hour 15	Hour 16	Hour 24
104	0.9152	0.9201	0.9172	0.9185	0.9183	0.9572
105	0.9091	0.9161	0.9152	0.9167	0.9164	0.9399
106	0.9015	0.9091	0.9052	0.9066	0.9061	0.9522
107	**0.8953**	0.9211	**0.8952**	**0.8865**	**0.8997**	0.9215
108	0.9091	0.9158	0.9336	0.9356	0.9356	0.9293
109	0.9096	0.9161	0.9512	0.9522	0.9522	0.9261
110	0.9153	0.9211	0.9657	0.9626	0.9631	0.9213
112	**0.8987**	0.9101	0.9366	0.9377	0.9372	0.9029

TABLE VII
LINE MAINTENANCE SCHEDULE WITH TRANSMISSION AND VOLTAGE CONSTRAINTS (CASE 2)

Line	Hour (1-24)																							
	1					6						12						18						24
1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
4	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
6	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1

In Case 2, the effect of voltage constraints on the maintenance schedule is considered. In addition of 110.6 MW at bus 42 which have been interrupted, 205 MVAR of recallable contracts at bus 59 have been interrupted at hours 14 and 15. The transmission provider has to purchase reactive power of 138 MVAR at hour 1 and 187 MVAR at hours 14, 15 and 16 from bus 89. System maintenance cost when voltage limits are not considered (Case 1) is $4655.64. As we impose voltage and transmission limits on the scheduling problem, a solution which satisfies both voltage and transmission limits is reached. The maintenance cost increases to $8244.08 as compared to $4655.64 when voltages were not considered. The reason that the cost is increased is that the maintenance schedule of lines 4 and 6

TABLE VIII
BUS VOLTAGES WITH VOLTAGE CONSTRAINTS IN LOCAL AREA (CASE 2)

Bus No	Voltage (p.u.)					
	Hour 1	Hour 2	Hour 14	Hour 15	Hour 16	Hour 24
104	0.9682	0.9737	0.9715	0.9723	0.9723	0.9222
105	0.9638	0.9701	0.9696	0.9706	0.9706	0.9211
106	0.9477	0.9548	0.9510	0.9521	0.9521	0.9106
107	0.9379	0.9471	0.9389	0.9404	0.9404	0.9010
108	0.9564	0.9625	0.9800	0.9809	0.9809	0.9375
109	0.9539	0.9598	0.9845	0.9854	0.9854	0.9449
110	0.9517	0.9565	1.0004	1.0011	1.0011	0.9690
112	0.9253	0.9327	0.9721	0.9732	0.9732	0.9421

which are the two longest lines are shifted to hours 5 and 6 (see Table B3). Table VIII shows corrected bus voltages in Case 2.

V. CONCLUSIONS

The paper presents a decomposition approach based on the duality theory for the short-term transmission line maintenance scheduling with transmission and voltage constraints. The formulation consists of a master program and sub-problems which consist of two independent programs. In the master problem, maintenance problem is solved and in sub-problems, transmission and voltage problems are solved independently.

The test results on the modified IEEE 118-bus system demonstrate that limits on transmission and voltage affect line maintenance scheduling and increase the maintenance cost. The cost-tradeoffs and constraints that are involved in evaluating the impact of line maintenance schedule are solved based on the proposed method. Introducing a minimum loss of revenue as one of the objective function, the proposed method is flexible enough to accommodate various pricing schemes.

APPENDIX A

$C'_{k\tau}$	Transmission maintenance cost per-line in the right-away k at hour τ.
$c_{i\tau}$	Loss of revenue per-MW at bus i and hour τ due to real power interruption.
$c'_{i\tau}$	Loss of revenue per-MVAR at bus i and hour τ due to reactive power interruption or purchasing reactive power from ancillary services.
$\boldsymbol{r}_{i\tau}$	Real power interruption at bus i and hour τ; in vector form is $\boldsymbol{r}$; maximum value is $\overline{r}$.
$s_{i\tau}$	Reactive power interruption or ancillary services at bus i and hour τ; in vector form is $\boldsymbol{s}$; maximum value is $\overline{s}$.
$\overline{N}_k$	Number of lines available in the right-of-way k.
M_k	Number of lines with maintenance in the right-of-way k.
$y_{k\tau}$	Line maintenance status, 0 if M_k lines in the right-of-way k are off-line for maintenance.
$Y^{\text{off}}_{k(\tau-1)}$	Number of weeks at hour $\tau - 1$ that lines in the right-of-way k have been on maintenance.
e'_k	Earliest period to begin maintenance of line k.
l'_k	Latest period to begin maintenance of line k.
d'_k	Duration of maintenance for line k.
$\overline{\boldsymbol{f}}_k$	Active power capacity of line k; in vector form is $\overline{\boldsymbol{f}}$.
f^k_τ	Active power flow on link k at hour τ; in vector form is $\boldsymbol{f}$.
$\boldsymbol{g}$	Vector of power generation for each unit at hour τ.
$\boldsymbol{d}$	Vector of hourly bus loads at hour τ.
$\boldsymbol{S}$	Node-branch incidence matrix.
$\boldsymbol{B}$	Susceptance matrix.
γ^k	Susceptance of branch k; in vector form is γ.
θ_i	Voltage angle associated with node i; in vector form is θ.

APPENDIX B

TABLE B1
HOURLY TOTAL TRANSACTIONS

τ	P MW	Q MVAR	τ	P MW	Q MVAR	τ	P MW	Q MVAR
1	4242	2969.4	9	4950	3465.0	17	4351	3045.7
2	3916	2741.2	10	5438	3806.6	18	4786	3350.2
3	3698	2588.6	11	5385	3769.5	19	4895	3426.5
4	3589	2512.3	12	5276	3693.2	20	4950	3465.0
5	3481	2436.7	13	5112	3578.4	21	4895	3426.5
6	3484	2438.8	14	5003	3502.1	22	4786	3350.2
7	3589	2512.3	15	4732	3312.4	23	4732	3312.4
8	3807	2664.9	16	4406	3084.2	24	4732	3312.4

TABLE B2
PARAMETERS OF TRANSMISSION LINES IN MAINTENANCE AREA

Line	Bus	Bus	R (pu)	X (pu)	Length (km)
1	3	5	0.1080	0.0142	150
2	5	6	0.0540	0.0071	70
3	6	7	0.0208	0.0027	50
4	8	9	0.0305	0.5810	580
5	3	12	0.0484	0.1600	200
6	9	10	0.0322	0.6150	600
7	4	11	0.0688	0.0087	100
8	5	11	0.0682	0.0087	100
9	11	12	0.0196	0.0025	50
10	2	12	0.0616	0.0079	80

TABLE B3
LINE MAINTENANCE COST

τ	$C'_{k\tau}$ (\$/km)	τ	$C'_{k\tau}$ (\$/km)	τ	$C'_{k\tau}$ (\$/km)
1	3.5	9	1.152	17	1.152
2	3.5	10	1.152	18	2.3
3	3.5	11	1.152	19	2.3
4	3.5	12	1.152	20	2.3
5	3.5	13	1.152	21	2.3
6	3.5	14	1.152	22	2.3
7	2.3	15	1.152	23	3.5
8	2.3	16	1.152	24	3.5

TABLE B4
RECALLABLE CONTRACTS

Bus	$\bar{r}+j\,\bar{s}$(%)*	c_{it} ($/MWh)	c'_{it} $/MVAR)
15	2.12	11.87	1.18
42	2.26	8.40	0.84
49	2.05	16.21	1.62
54	2.66	18.47	1.84
59	6.53	8.98	0.90
74	1.6	27.88	2.79
80	3.06	21.40	2.14
89	3.84	17.96	1.80
112	1.60	19.28	1.93
116	4.34	9.53	0.95

*) percent of total hourly load in Table B1

TABLE B5
ANCILLARY SERVICES FOR REACTIVE POWER

Bus	$\bar{s}$(MVAR)	c'_{it} ($/MVAR)
65	277	1.3
66	277	1.2
69	465	1.1
80	390	1.0
89	550	0.8
100	230	0.9

REFERENCES

[1] D. Shirmohammadi *et al.*, "Cost of transmission transactions: Introduction," *IEEE Trans. on Power Systems*, vol. 6, no. 3, pp. 1006–1016, August 1991.
[2] D. Shirmohammadi *et al.*, "Evaluation of transmission network capacity use for wheeling transaction," *IEEE Trans. on Power Systems*, vol. 4, no. 4, pp. 1405–1413, October 1989.
[3] H. H. Happ, "Cost of wheeling methodologies," *IEEE Trans. on Power Systems*, vol. 9, no. 1, Feb. 1994.
[4] D. Shirmohammadi, X. V. Filho, B. Gorenstin, and M. V. P. Pereira, "Some fundamental technical concepts about cost based transmission pricing," *IEEE Trans. on Power Systems*, vol. 11, no. 2, May 1996.
[5] C. W. Yu and A. K. David, "Pricing transmission services in context of industry deregulation," *IEEE Trans. on Power Systems*, vol. 12, no. 1, May 1997.
[6] Federal Energy Regulatory Commission, "Capacity Reservation Open Access Transmission Tariffs,", NOPR, April 1996.
[7] National Electric Reliability Council, "Available Transfer Capability Definitions and Determination,", NERC Report, May 1996.
[8] L. Lasdon, *Optimization Theory for Large Systems*. New York: MacMillan, 1970.
[9] "IEEE reliability test system," *IEEE Trans. on Power Apparatus and Systems*, vol. PAS-98, no. 6, pp. 2047–2054, Nov./Dec. 1979.
[10] M. V. F. Pereira *et al.*, "A decomposition approach to automated generation/transmission expansion planning," *IEEE Trans. on Power Apparatus Systems*, vol. PAS-104, no. 11, November 1985.
[11] L. Chen and J. Toyoda, "Optimal generating unit maintenance scheduling for multi-area system with network constraints," *IEEE Trans. on Power System*, vol. 6, pp. 1168–1174, 1991.
[12] E. L. Silva *et al.*, "Transmission constrained maintenance scheduling of generating units: A stochastic programming approach," *IEEE Trans. on Power Systems*, vol. 10, no. 2, pp. 695–701, May 1995.
[13] M. K. C. Marwali, S. M. Shahidehpour, and V. C. Ramesh, "A decomposition approach to generation maintenance scheduling with network constraints," *APC Proc.*, vol. 59-I, pp. 110–115, April 1997.
[14] M. K. C. Marwali and S. M. Shahidehpour, "Integrated generation and transmission maintenance scheduling with network constraints," *IEEE Trans. on Power Systems*, vol. 13, no. 3, pp. 1063–1068, August 1998.
[15] J. Qiu and S. M. Shahidehpour, "A new approach for minimizing power losses and improving voltage profile," *IEEE Transactions on Power Systems*, vol. PWRS-2, no. 2, pp. 287–295, May 1987.

M. K. C. Marwali received his Ph.D. degree in electrical engineering from the Illinois Institute of Technology in 1997. He is currently with ABB in Santa Clara, CA. He has written several papers on power systems scheduling and its coordination with long-term planning.

S. M. Shahidehpour is a Professor in the ECE Department and Dean of Graduate College at the Illinois Institute of Technology. He is one the Editors of the IEEE TRANSACTIONS ON POWER SYSTEMS and serves on the Organizing Committee of the PICA. He can be reached at ms@iit.edu.