TUTORIAL

BENDERS DECOMPOSITION IN RESTRUCTURED POWER SYSTEMS

Mohammad Shahidehpour and Yong Fu
Electric Power and Power Electronics Center
Illinois Institute of Technology
Chicago, Il 60616
{ms, fuyong}@iit.edu

1. Introduction

It is apparent that the power system restructuring provides a major forum for the application of decomposition techniques to coordinate the optimization of various objectives among self-interested entities. These entities include power generators (GENCO), transmission providers (TRANSCO), and distribution companies (DISCO). Consider a decomposition example when individual GENCOs optimize their annual generating unit maintenance schedule based on their local constraints such as available fuel, emission, crew, and seasonal load profile. The GENCO's optimization intends to maximize the GENCO's payoff in a competitive environment. Individual GENCOs submit their maintenance schedule to the ISO which examines the proposed schedule to minimize the loss of load expectation while maintaining the transmission security based on the available transfer capacity, and forced and scheduled outages of power system components. The ISO could return then proposed schedule to designated GENCOs in case the operating constraints would be violated. The ISO's rejection of the proposed schedule could include a suggestion (Benders cut) for revising the proposed maintenance schedule that would satisfy GENCOs' and the ISO's constraints.

Earlier in the 1960-1970, many of the decomposition techniques were motivated by inability to solve large-scale centralized problems with the available computing power of that time. The dramatic improvement in computing technology since then allowed power engineers to solve very large problems easily. Consequently, interest in decomposition techniques dropped dramatically. However, now there is an increasingly important class of optimization problems in restructured power systems for which decomposition techniques are becoming most relevant.

In principle, one may consider the optimization of a system of independent entities by constructing a large-scale mathematical program and solving it centrally (e.g., through the ISO) using currently available computing power and solution techniques. In practice, however, this is often impossible. In order to solve a problem centrally, one needs the complete information on local objective functions and constraints. As these entities are separated geographically and functionally, this information may be unattainable or prohibitively expensive to retrieve. More importantly, independent entities may be unwilling to share or report on their propriety information as it is not incentive compatible to do so; i.e., these entities may have an incentive to misrepresent their true preferences. In order to optimize certain objectives in restructured power systems, one must turn to the coordination aspects of decomposition. Specifically, with limited information one must coordinate entities to reach an optimal solution. The goal will be to coordinate the entities by optimizing a certain objective (such as finding equilibrium resource price) while satisfying local and system constraints.

One of the commonly used decomposition techniques in power systems is Benders decomposition. J. F. Benders introduced the Benders decomposition algorithm for solving large-scale mixed-integer programming (MIP) problems. Benders decomposition has been successfully applied to take advantage of underlying problem structures for various optimization problems, such as restructured power systems operation and planning, electronic packaging and network design, transportation, logistics, manufacturing, military applications, and warfare strategies.

In applying Benders decomposition, the original problem will be decomposed into a master problem and several subproblems. Generally, the master-program is an integer problem and subproblems are the linear programs. The lower bound solution of the master problem may involve fewer constraints. The subproblems will examine the solution of the master problem to see if the solution satisfies the remaining constraints. If the subproblems are feasible, the upper bound solution of the original problem will be calculated while forming a new objective function for the further optimization of the master problem solution. If any of the subproblems is infeasible, an infeasibility cut representing the least satisfies constraint will be introduced to the master problem. Then, a new lower bound solution of the original problem will be obtained by re-calculating the master problem with more constraints. The final solution based on the Benders decomposition algorithm may require iterations between the master problem and subproblems. When the upper bound and the lower bound are sufficiently close, the optimal solution of the original problem will be achieved.

Fig. 1.1 depicts the hierarchy for calculating security-constrained unit commitment (SCUC) which is based on the existing set up (GENCOs and TRANSCOs as separate entities) in restructured power systems. The hierarchy utilizes a Benders decomposition which decouples the SCUC into a master problem (optimal generation scheduling) and network security check subproblems. The output of the master problem is the on/off state of units which are examined in the subproblem for satisfying the network constraints. The network violations are formulated in the form of Benders cuts which ate are added to the optimal generation scheduling formulation for re-calculating the original unit commitment solution.

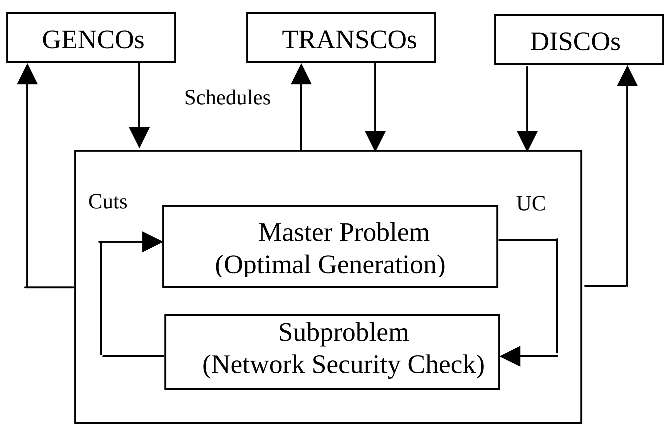

Fig. 1.1 ISO and market participants

Other applications of Benders decomposition to security-constrained power systems include:

- Generating Unit Planning
- Transmission Planning
- Optimal Generation Bidding and Valuation
- Reactive Power Planning
- Optimal Power Flow
- Hydro-thermal Scheduling
- Generation Maintenance Scheduling
- Transmission Maintenance Scheduling
- Long-term Fuel Budgeting and Scheduling
- Long-term Generating Unit Scheduling and Valuation

In order to discuss the applications of Benders decomposition to power systems, we review in the following the subject of duality in linear programs.

2. Primal and Dual Linear Programs

In this section, the relationship between primal and dual problems and the related duality theorems are discussed. Every linear program (LP), called the *primal problem* can be equivalently expressed in another LP form called the *dual problem*. The primal problem can be expressed in matrix notation as follows:

$$\begin{aligned} &\textit{Minimize } z = \mathbf{c}^{\mathbf{T}}\mathbf{x} \\ &s.t. \qquad \mathbf{Ax} \geq \mathbf{b} \\ &\qquad\qquad \mathbf{x} \geq \mathbf{0} \end{aligned} \qquad \text{Primal} \qquad (2.1)$$

where $\mathbf{c}$ and $\mathbf{x}$ are n-vector, $\mathbf{b}$ is an m-vector and $\mathbf{A}$ is an *m x n* matrix. The linear function $\mathbf{c}^{\mathbf{T}}\mathbf{x}$ is called *objective function*. The linear inequalities are called *constraints* and they form a *feasible region* for minimizing the objective function. The solution elements of the primal problem in the feasible region is written as $\{\mathbf{x} \in \mathbf{R}^{\mathbf{n}} \mid \mathbf{Ax} \geq \mathbf{b}, \mathbf{x} \geq \mathbf{0}\}$. Also, its corresponding dual problem is defined as:

$$\begin{aligned} &\textit{Maximize } z = \mathbf{b}^{\mathbf{T}}\mathbf{y} \\ &s.t. \qquad \mathbf{A}^{\mathbf{T}}\mathbf{y} \leq \mathbf{c} \\ &\qquad\qquad \mathbf{y} \geq \mathbf{0} \end{aligned} \qquad \text{Dual} \qquad (2.2)$$

The number of inequalities in the primal problem becomes the number of variables in the dual problem. Correspondingly, the number of variables in the primal problem becomes the number of inequalities in the dual problem. Hence the dual problem differs in dimensions from the primal problem.

It is typically easier to solve an LP with fewer constraints. Since the primal problem has m constraints while the dual problem has n constraints, this generates the following rule of thumb: Solve the LP problem that has the fewer number of constraints. For instance, solve the primal problem if m<n, but solve the dual problem if m>n. The relationship between primal and dual problems is listed in Table 2.1.

Table 2.1

Primal (or Dual)		**Dual (or Primal)**	
Objective	*Max z*	*Min w*	Objective
Variable (n)	≥ 0	≥	Constraints (n)
	≤ 0	≤	
	Unlimited	=	
Constraints (m)	≤	≥ 0	Variable (m)
	≥	≤ 0	
	=	Unlimited	
Right-side vector of constraints		Coefficients of variables in objective function	
Coefficients of variables in objective function		Right-side vector of constraints	

Example 2.1:

Primal problem

$$\begin{aligned} &Max\ z = 5x_1 + 4x_2 + 6x_3 \\ &S.t. \quad x_1 + 2x_2 \geq 2 \\ &\quad x_1 + x_3 \leq 3 \\ &\quad -3x_1 + 2x_2 + x_3 \leq -5 \\ &\quad x_1 - x_2 + x_3 = 1 \\ &\quad x_1 \geq 0,\ x_2 \leq 0,\ x_3\ unlimited \end{aligned}$$

→

Dual problem

$$\begin{aligned} &Min\ w = 2y_1 + 3y_2 - 5y_3 + y_4 \\ &S.t. \quad y_1 + y_2 - 3y_3 + y_4 \geq 5 \\ &\quad 2y_1 + 2y_3 - y_4 \leq 4 \\ &\quad y_2 + y_3 + y_4 = 6 \\ &\quad y_1 \leq 0,\ y_2, y_3 \geq 0,\ y_4\ unlimited \end{aligned}$$

3. Basic Model of Benders Decomposition

A mixed-integer program has the following form:

$$\begin{aligned} &Minimize\ z = \mathbf{c}^{\mathbf{T}}\mathbf{x} + \mathbf{d}^{\mathbf{T}}\mathbf{y} \\ &s.t. \quad \mathbf{Ay} \geq \mathbf{b} \\ &\quad \mathbf{Ex} + \mathbf{Fy} \geq \mathbf{h} \\ &\quad \mathbf{x} \geq \mathbf{0},\ \mathbf{y} \in \mathbf{S} \end{aligned} \qquad \text{P1} \qquad (3.1)$$

where,

$\mathbf{A}$: $m \times n$ matrix,

$\mathbf{E}$: $q \times p$ matrix,

$\mathbf{F}$: $q \times n$ matrix,

$\mathbf{x}, \mathbf{c}$: p vectors,

$\mathbf{y}, \mathbf{d}$: n integer vector,

$\mathbf{b}$: m vector,

$\mathbf{h}$: q vector,

$\mathbf{S}$: an arbitrary subset of E^p with integral-valued components

Since $\boldsymbol{x}$ is continuous and $\boldsymbol{y}$ is integer, (P1) is a mixed-integer problem. If $\boldsymbol{y}$ values are fixed, (P1) is linear in $\boldsymbol{x}$. Hence, (3.1) is written as:

$$\underset{\mathbf{y} \in \mathbf{R}}{\text{Minimize}} \left\{ \mathbf{d}^{\mathbf{T}}\mathbf{y} \middle| \mathbf{Ay} \geq \mathbf{b} + Min\left\{ \mathbf{c}^{\mathbf{T}}\mathbf{x} \mid \mathbf{Ex} \geq \mathbf{h} - \mathbf{Fy}, \mathbf{x} \geq \mathbf{0} \right\} \right\} \qquad (3.2)$$

where,

$$\mathbf{R} = \left\{ \mathbf{y} \middle| \text{ there exists } \mathbf{x} \geq \mathbf{0} \text{ such that } \mathbf{Ex} \geq \mathbf{h} - \mathbf{Fy},\ \mathbf{Ay} \geq \mathbf{b},\ \mathbf{y} \in \mathbf{S} \right\} \qquad (3.3)$$

So, the original problem can be decoupled into a master problem (MP) and a subproblem (SP).

We begin with solving the following master problem MP1 (3.4):

$$\begin{aligned} &Minimize\ z_{lower} \\ &s.t. \quad z_{lower} \geq \mathbf{d}^{\mathbf{T}}\mathbf{y} \\ &\quad \mathbf{Ay} \geq \mathbf{b} \\ &\quad \mathbf{y} \in \mathbf{S} \end{aligned} \qquad \text{MP1} \qquad (3.4)$$

We use z, instead of $\mathbf{d}^{\mathbf{T}}\mathbf{y}$, as the objective function. The inner part of minimization (3.2) is a subproblem SP1 rewritten as

Primal subproblem (SP1)

$$\begin{aligned} &\textit{Minimize}\ \ \mathbf{c}^{\mathbf{T}}\mathbf{x} \\ &\text{s.t.}\quad \mathbf{Ex} \geq \mathbf{h} - \mathbf{F}\hat{\mathbf{y}} \qquad \text{SP1} \\ &\qquad\ \ \mathbf{x} \geq \mathbf{0} \end{aligned} \tag{3.5}$$

Also SP2 is the dual subproblem of SP1 given as

$$\begin{aligned} &\textit{Maximize}\quad (\mathbf{h} - \mathbf{F}\hat{\mathbf{y}})^{\mathbf{T}}\ \mathbf{u} \\ &\text{s.t.}\qquad \mathbf{E}^{\mathbf{T}}\mathbf{u} \leq \mathbf{c} \qquad \text{SP2} \\ &\qquad\qquad \mathbf{u} \geq \mathbf{0} \end{aligned} \tag{3.6}$$

where $\hat{\mathbf{y}}$ is the solution of the master problem.

The flowchart for the Benders decomposition is as shown in Figure 4.1.

4. Solution Steps for the Benders Cut Algorithm

Step 1. Solve MP1 (3.4) and obtain an initial lower bound solution given as $\hat{z}_{lower}$ at $\hat{\mathbf{y}}$. If MP1 is infeasible so will be the original problem P1. If MP1 is unbounded, set $\hat{z}_{lower} = \infty$ in (3.4) for an arbitrary value of $\hat{\mathbf{y}}$ in S, and go to step 2.

Step 2. Solve SP1 (3.5) or SP2 (3.6). An upper bound solution of the original problem P1, in terms of SP2, is $\hat{z}_{upper} = \mathbf{d}^{\mathbf{T}}\hat{\mathbf{y}} + (\mathbf{h} - \mathbf{F}\hat{\mathbf{y}})^{\mathbf{T}}\hat{\mathbf{u}}^{\mathbf{p}}$ for the optimal dual solution $\hat{\mathbf{u}}^{\mathbf{P}}$. In terms of SP1, $\hat{z}_{upper} = \mathbf{d}^{\mathbf{T}}\hat{\mathbf{y}} + \mathbf{c}^{\mathbf{T}}\hat{\mathbf{x}}$ is the upper bound solution of the original problem P1 for $\hat{\mathbf{x}}$.

- If $\left|\hat{z}_{upper} - \hat{z}_{lower}\right| \leq \varepsilon$ for P1, then stop the process. Otherwise, generate a new constraint $z_{lower} \geq \mathbf{d}^{\mathbf{T}}\mathbf{y} + (\mathbf{h} - \mathbf{F}\mathbf{y})^{\mathbf{T}}\hat{\mathbf{u}}^{\mathbf{p}}$ **(feasibility cut)** for MP2 (3.8) and go to step 3.
- If SP2 is unbounded, which means that SP1 is infeasible, then introduce a new cut $(\mathbf{h} - \mathbf{F}\mathbf{y})^{\mathbf{T}}\hat{\mathbf{u}}^{\mathbf{r}} \leq \mathbf{0}$ **(infeasibility cut)** for MP2 (3.8). In this case, we will first calculate $\mathbf{u}^{\mathbf{r}}$ from (3.7) to form the infeasibility cut and then go to step 3. We use a new SP1 (3.7), feasibility check subproblem to calculate $\mathbf{u}^{\mathbf{r}}$ in SP2.

$$\begin{aligned} &\text{Minimize}\ \ \mathbf{1}^{\mathbf{T}}\mathbf{s} \\ &\text{St.}\quad \mathbf{Ex} + \mathbf{Is} \geq \mathbf{h} - \mathbf{F}\hat{\mathbf{y}} \qquad \rightarrow \quad \mathbf{u}^{\mathbf{r}} \\ &\qquad\quad \mathbf{x} \geq \mathbf{0}, \mathbf{s} \geq \mathbf{0} \end{aligned} \tag{3.7}$$

  where $\mathbf{1}$ is the unit vector.

- If SP2 is infeasible, the original problem P1 will have either no feasible solution or an unbounded solution. Stop the process.

Step 3. Solve MP2 to obtain a new lower bound solution $\hat{z}_{lower}$ with respect to $\hat{\mathbf{y}}$ for the original problem P1. In the following MP2 formulation, we use either the *feasibility cut* (second constraint) or the *infeasibility cut* (third constraint) as discussed in Step 2.

$$\begin{aligned} & \textit{Minimize } z_{lower} \\ & s.t. \quad \mathbf{Ay} \ge \mathbf{b} \\ & \qquad \mathbf{z_{lower}} \ge \mathbf{d}^{\mathbf{T}}\mathbf{y} + (\mathbf{h} - \mathbf{Fy})^{\mathbf{T}}\mathbf{u_i^p},\ i = 1,\ldots,n_p \qquad \text{MP2} \qquad (3.8) \\ & \qquad (\mathbf{h} - \mathbf{Fy})^{\mathbf{T}}\mathbf{u}_i^{\mathbf{r}} \le \mathbf{0},\ i = 1,\ldots,n_r \\ & \qquad \mathbf{y} \in \mathbf{S} \end{aligned}$$

- Then go back to step 2 for solving the subproblem SP again.
- If MP2 is unbounded, specify $\hat{z}_{lower} = \infty$ with $\hat{\mathbf{y}}$ as an arbitrary element of $\mathbf{S}$. Return to step 2.
- If MP2 is infeasible, so will be the original problem P1. Stop the process.

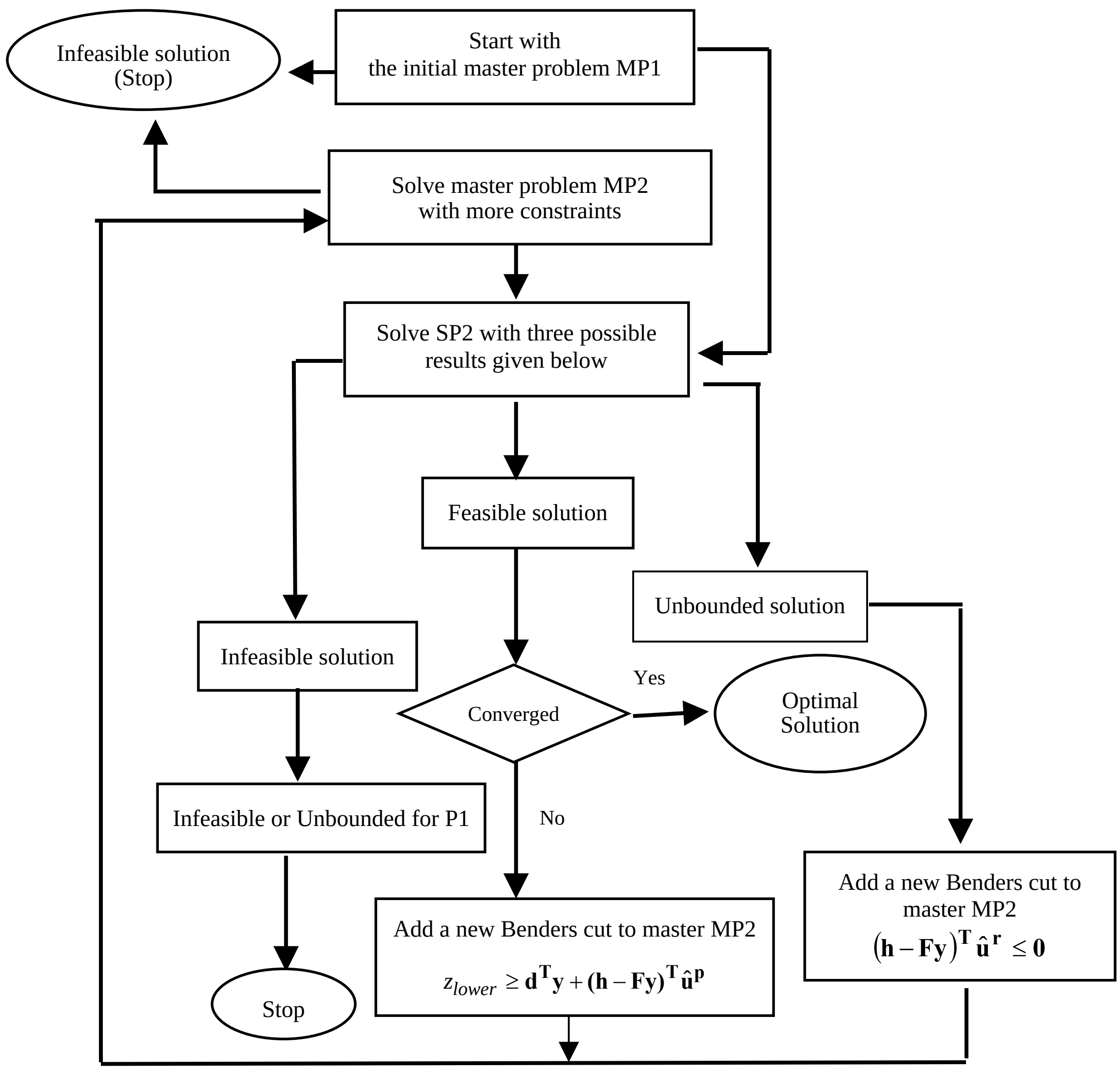

Figure 4.1: Flowchart of Benders Decomposition

Example 4.1

The original problem is

$$\begin{aligned} &Min\ x + y \\ &S.t.\quad 2x + y \geq 3 \\ &\qquad x \geq 0,\ y \in \{-5,-4,\ldots,3,4\} \end{aligned}$$

$\mathbf{c^T} = [1]$ $\mathbf{d^T} = [1]$ $\mathbf{E} = [2]$ $\mathbf{F} = [1]$ $\mathbf{h} = [3]$

Iteration 1: Form MP1.

$$\begin{aligned} &Min\ z_{lower} \\ &S.t.\quad z_{lower} \geq y \\ &\qquad y \in \{-5,-4,\ldots,3,4\} \end{aligned}$$

The lower bound optimal solution of the original problem is $\hat{z}_{lower} = -5$ when $\hat{y} = -5$.

Form the SP1 subproblem.

$$\begin{aligned} &Min\ x \\ &S.t.\quad 2x \geq 3 - \hat{y} \\ &\qquad x \geq 0 \end{aligned}$$

or, Form the SP2 subproblem.

$$\begin{aligned} &Max\ (3-\hat{y})u \\ &S.t.\quad 2u \leq 1 \\ &\qquad u \geq 0 \end{aligned} \quad \overset{\hat{y}=-5}{\Rightarrow} \quad \begin{aligned} &Max\ \ 8u \\ &S.t.\quad 2u \leq 1 \\ &\qquad u \geq 0 \end{aligned}$$

We choose to solve SP2 and get the optimal solution equal to 4 at $u = \frac{1}{2}$. Thus, the upper bound optimal solution of the original problem is $\hat{z}_{upper} = \hat{y} + 4 = -5 + 4 = -1$. We continue with the next iteration because $\hat{z}_{upper} = -1 > \hat{z}_{lower} = -5$.

Iteration 2: Form MP2 with a new constraint $z \geq y + (3-y) * \frac{1}{2}$.

$$\begin{aligned} &Min\ z_{lower} \\ &S.t.\quad z_{lower} \geq y \\ &\qquad z_{lower} \geq y + (3-y) * \frac{1}{2} \\ &\qquad y \in \{-5,-4,\ldots,3,4\} \end{aligned} \quad \Rightarrow \quad \begin{aligned} &Min\ z_{lower} \\ &S.t.\quad z_{lower} \geq y \\ &\qquad z_{lower} \geq \frac{3}{2} + \frac{1}{2}y \\ &\qquad y \in \{-5,-4,\ldots,3,4\} \end{aligned}$$

The new lower bound optimal solution of the original problem is $\hat{z}_{lower} = -1$ for $\hat{y} = -5$.

Solve SP2.

$$\begin{array}{lcl} Max\ (3-\hat{y})u & & Max\ \ 8u \\ S.t.\quad 2u \le 1 & \overset{\hat{y}=-5}{\Rightarrow} & S.t.\quad 2u \le 1 \\ \qquad u \ge 0 & & \qquad u \ge 0 \end{array}$$

So, the upper bound optimal solution of the original problem is $\hat{z}_{upper} = \hat{y}+4 = -5+4 = -1$. The process has converged because $\hat{z}_{upper} = \hat{z}_{lower} = -1$.

5. Alternative Form of Benders Cuts

Benders cuts were expressed as

$$\begin{aligned} &z \ge \mathbf{d^T y} + \mathbf{(h - Fy)^T u}_i^{\mathbf{p}},\ i = 1,\ldots,n_p \\ &\left(\mathbf{h} - \mathbf{Fy}\right)^{\mathbf{T}} \mathbf{u}_i^{\mathbf{r}} \le \mathbf{0},\ i = 1,\ldots,n_r \end{aligned} \tag{5.1}$$

Alternatively, (5.1) could be represented as (5.2) in which the first equation is the feasibility cut and the second one is the infeasibility cut.

$$\begin{aligned} &z \ge \mathbf{d^T y} + w(\hat{\mathbf{y}})_i - (\mathbf{y} - \hat{\mathbf{y}})^{\mathbf{T}} \mathbf{F^T u_i^P},\ i = 1,\ldots,n_p \\ &v(\hat{\mathbf{y}})_i - (\mathbf{y} - \hat{\mathbf{y}})^{\mathbf{T}} \mathbf{F^T u_i^r} \le \mathbf{0},\ i = 1,\ldots,n_r \end{aligned} \tag{5.2}$$

where,

$w(\hat{\mathbf{y}})$ Optimal solution of SP1 (3.5)

$v(\hat{\mathbf{y}})$ Optimal solution of the feasibility check subproblem (3.7)

The Benders cut $z \ge \mathbf{d^T y} + w(\hat{\mathbf{y}}) + (\mathbf{y} - \hat{\mathbf{y}})^{\mathbf{T}} \mathbf{F^T u^P}$ indicates that we decrease the objective value of the original problem by updating $\mathbf{y}$ from $\hat{\mathbf{y}}$ to a new value. The dual multiplier vector $\mathbf{u^P}$ represents the incremental change in the optimal objective. Similarly, the Benders cut $v(\hat{\mathbf{y}}) + (\mathbf{y} - \hat{\mathbf{y}})^{\mathbf{T}} \mathbf{F^T u^r} \le \mathbf{0}$ indicates that we update $\hat{\mathbf{y}}$ to a new value to eliminate constraint violations in SP1 based on $\hat{\mathbf{y}}$ given in the master problem. The dual multiplier vector $\mathbf{u^r}$ represents the incremental change in the total violation.

Example 5.1

We use the Form 2 of Benders cuts to solve the following example.

$$\begin{aligned} &Min\ \ x_1 + 3x_2 + y_1 + 4y_2 \\ &S.t.\quad -2x_1 - x_2 + y_1 - 2y_2 \ge 1 \\ &\qquad\quad 2x_1 + 2x_2 - y_1 + 3y_2 \ge 1 \\ &\qquad\quad x_1, x_2 \ge 0, y_1, y_2 \ge 0 \end{aligned}$$

In general, since we had,

$$\begin{aligned} &Minimize\ z = \mathbf{c^T x} + \mathbf{d^T y} \\ &s.t.\qquad \mathbf{Ay} \ge \mathbf{b} \\ &\qquad\qquad \mathbf{Ex} + \mathbf{Fy} \ge \mathbf{h} \\ &\qquad\qquad \mathbf{x} \ge \mathbf{0},\ \mathbf{y} \in \mathbf{S} \end{aligned}$$

Accordingly, for the above example,

$$\mathbf{c^T} = [1\ 3] \qquad \mathbf{d^T} = [1 \quad 4] \quad \mathbf{E} = \begin{bmatrix} -2 & -1 \\ 2 & 2 \end{bmatrix} \qquad \mathbf{F} = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} \qquad \mathbf{h} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Iteration 1: Solve MP1

$$\begin{aligned} &Min \quad z_{lower} \\ &S.t. \quad z_{lower} \ge y_1 + 4y_2 \\ &\qquad y_1 \ge 0,\ y_2 \ge 0 \end{aligned}$$

which results in $\hat{y}_1 = 0, \quad \hat{y}_2 = 0, \quad \hat{z}_{lower} = 0$. We use the feasibility check subproblem (3.7) because SP2 is unbounded at $\hat{y}_1 = 0, \quad \hat{y}_2 = 0$.

$$\begin{aligned} &Min \quad s_1 + s_2 \\ &St. \quad -2x_1 - x_2 + s_1 \qquad \ge 1 - \hat{y}_1 + 2\hat{y}_2 \qquad\qquad u_1 \\ &\qquad 2x_1 + 2x_2 \qquad + s_2 \ge 1 + \hat{y}_1 - 3\hat{y}_2 \qquad\qquad u_2 \\ &\qquad\quad x_1, x_2 \ge 0, s_1, s_2 \ge 0 \end{aligned}$$

The optimal solution is 1.5 and its dual multipliers are $\hat{u}_1 = 1.0, \hat{u}_2 = 0.5$. The Benders cut is $1.5 - 0.5*(y_1 - \hat{y}_1) + 0.5*(y_2 - \hat{y}_2) \le 0 \Rightarrow y_1 - y_2 \ge 3$ at $\hat{y}_1 = 0, \quad \hat{y}_2 = 0$.

Iteration 2: The new master problem MP2 is

$$\begin{aligned} &Min\ z_{lower} \\ &S.t. \quad z_{lower} \ge y_1 + 4y_2 \\ &\qquad y_1 - y_2 \ge 3 \\ &\qquad y_1, y_2 \ge 0 \end{aligned}$$

Hence, the new lower bound optimal solution of the original problem is $\hat{z}_{lower} = 3$ for $\hat{y}_1 = 3, \quad \hat{y}_2 = 0$. We form the primal subproblem SP1 as

$$\begin{aligned} &Min \quad x_1 + 3x_2 \\ &St. \quad -2x_1 - x_2 \ge 1 - \hat{y}_1 + 2\hat{y}_2 \qquad\qquad u_1 \\ &\qquad 2x_1 + 2x_2 \ge 1 + \hat{y}_1 - 3\hat{y}_2 \qquad\qquad u_2 \\ &\qquad\quad x_1, x_2 \ge 0 \end{aligned}$$

Here SP1 is feasible with an optimal solution equal to 6 and dual multipliers equal to $\hat{u}_1 = 2.0, \hat{u}_2 = 2.5$. The feasibility cut is $z_{lower} \ge y_1 + 4y_2 + 6 + 0.5*(y_1 - \hat{y}_1) - 3.5*(y_2 - \hat{y}_2)$.

So $z_{lower} \ge 4.5 + 1.5y_1 + 0.5y_2$. Accordingly, the upper bound solution of the original problem is $\hat{z}_{upper} = \hat{y}_1 + 4\hat{y}_2 + 6 = 3 + 6 = 9$. We will continue the process because $\hat{z}_{upper} = 9 > \hat{z}_{lower} = 3$.

Iteration 3: Add $z_{lower} \ge 4.5 + 1.5y_1 + 0.5y_2$ to MP2. So,

$Min\ z_{lower}$

$$S.t.\quad z_{lower} \geq y_1 + 4y_2$$
$$z_{lower} \geq 4.5 + 1.5y_1 + 0.5y_2$$
$$y_1 - y_2 \geq 3$$
$$y_1, y_2 \geq 0$$

Hence, the new lower bound solution of the original problem is $\hat{z}_{lower} = 9$ when $\hat{y}_1 = 3, \quad \hat{y}_2 = 0$.
Solve SP1. The optimal solution is 6 and $\hat{z}_{upper} = \hat{y}_1 + 4\hat{y}_2 + 6 = 3 + 6 = 9$. We terminate the iterative optimization process because $\hat{z}_{upper} = \hat{z}_{lower} = 9$.

6. Benders Decomposition for Security-Constrained Unit Commitment (SCUC)

In order to apply Benders decomposition to SCUC, we write the SCUC problem as a standard Benders formulation. The startup cost of unit *i* is expressed as $st_i\alpha_{it}$ where st_i is the startup cost and α_{it} is a binary variable that is equal to 1 if unit *i* is started up at hour t and is 0 otherwise. The shutdown cost is expressed similarly as $sd_i\beta_{it}$ where sd_i is the shutdown cost of unit *i* and β_{it} is a binary variable that is equal to 1 if unit *i* is shut down at hour t and is 0 otherwise. The production cost is proportional to the unit output power which is expressed as $c_i p_{it}$ where c_i is the cost coefficient of unit *i* and p_{it} is the generated power of unit *i* at hour t. Thus, the objective of SCUC is written as:

$$Min\ Z = \sum_{t=1}^{T}\sum_{i=1}^{NG} c_i p_{it} + st_i\alpha_{it} + sd_i\beta_{it} \tag{6.1}$$

A unit that is online can be shut down but not started up. Similarly, a unit that is offline can be started up but not shut down. This can be expressed as

$$\alpha_{it} - \beta_{it} = I_{it} - I_{i(t-1)}\,. \tag{6.2}$$

where I_{it} is a binary variable that is equal to 1 if unit *i* is online during hour t and is 0 otherwise.

For the first hour, the above constraint becomes $\alpha_{i1} - \beta_{i1} = I_{i1} - I_{i0}$ where I_{i0} is the initial state of unit i. Its value is 1 if unit i is online at the initial hour and is 0 otherwise. The minimum up/down time limits of a unit are given as:

$$\begin{aligned}
&\sum_{t}^{t+T_{i,\min}^{on}-1} I_{it} \geq \alpha_{it} * T_{i,\min}^{on} \quad (t = 1,\cdots, NT - T_{i,\min}^{on} + 1)\\
&\sum_{t}^{NT} I_{it} \geq \alpha_{it} * (NT - t + 1) \quad (t = NT - T_{i,\min}^{on} + 2,\cdots, NT)\\
&\sum_{t}^{t+T_{i,\min}^{off}-1} [1 - I_{it}] \geq \beta_{it} * T_{i,\min}^{off} \quad (t = 1,\cdots, NT - T_{i,\min}^{off} + 1)\\
&\sum_{t}^{NT} [1 - I_{it}] \geq \beta_{it} * (NT - t + 1) \quad (t = NT - T_{i,\min}^{off} + 2,\cdots, NT)
\end{aligned} \tag{6.3}$$

where $T_{i,\min}^{on}$ is the minimum up time of unit i and $T_{i,\min}^{off}$ is the minimum down time of unit i. For instance, if $T_{i,\min}^{on} = 2$ and $T_{i,\min}^{off} = 3$, Table 6.1 shows the relationship between variables α_{it}, β_{it} and I_{it}.

Table 6.1 Relationship among α_{it}, β_{it} and I_{it}

Hours	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
I	0	1	1	1	0	0	0	1	1	0	0
α	-	1	0	0	0	0	0	1	0	0	0
β	-	0	0	0	1	0	0	0	0	1	0

The additional constraints are given as

System reserve requirements

$$\sum_{i=1}^{NG} p_{i,\max} I_{it} \geq D_t + R_t \tag{6.4}$$

where NG is the number of units, D_t is the demand in hour t, and R_t is system reserve at hour t.

Hourly power demand

$$\sum_{i=1}^{NG} p_{it} = D_t \tag{6.5}$$

Thermal unit capacity constraint

$$P_{i,\min} I_{it} \leq p_{it} \leq P_{i,\max} I_{it} \tag{6.6}$$

Hourly network constraint

$$-PL_{km,\max} \leq f_{km,t}(\mathbf{I},\mathbf{p}) \leq PL_{km,\max} \tag{6.7}$$

where f_{km} is the power flow on the line extending from bus k to bus m and $PL_{km,\max}$ is the line capacity.

6.1 Solution Procedure

The detailed SCUC solution procedure is shown in Figure 6.1.

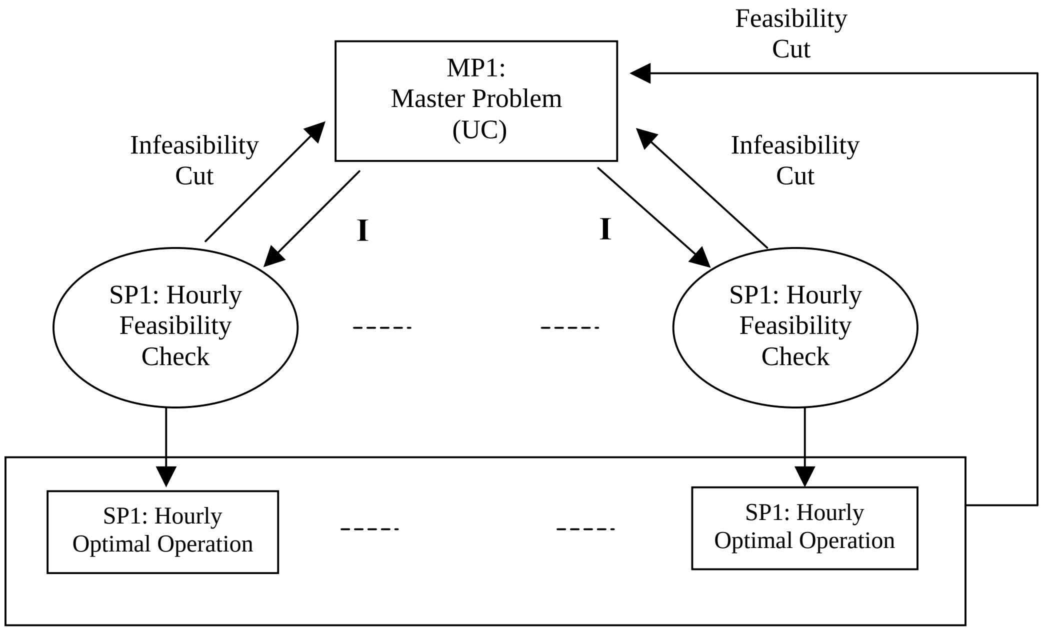

Figure 6.1 SCUC Algorithm

The initial SCUC master problem **(MP1)** is formulated as

$$Min \;\; Z_{lower}$$

$$Z_{lower} \geq \sum_{t=1}^{T} \sum_{i=1}^{NG} st_i \alpha_{it} + sd_i \beta_{it} \tag{6.8}$$

S.t.

additional constraints (6.2)-(6.4).

In this case, **SP1** consists of the following two processes as shown in Figure 6.1.

1. The initial solution of I_{it} is introduced to the *hourly feasibility check subproblem* to determine whether the initial solution satisfies network constraints. The hourly feasibility subproblem for minimizing load curtailments is written as follow:

$$Min \quad v_t^n = \Sigma r_k^n \tag{6.9}$$

S.t.

First Kirchoff's law - bus power balance

$$\mathbf{sf} + \mathbf{P_g} + \mathbf{r} = \mathbf{P_D} \tag{6.10}$$

Second Kirchoff's law - line power flow

$$f_{km} = \gamma_{km}(\theta_k - \theta_m) \tag{6.11}$$

$$\begin{aligned} \mathbf{P_g} &\leq \mathbf{P_{g,max}} \hat{\mathbf{I}} \qquad \overline{\boldsymbol{\lambda}} \\ -\mathbf{P_g} &\leq -\mathbf{P_{g,min}} \hat{\mathbf{I}} \qquad \underline{\boldsymbol{\lambda}} \end{aligned} \tag{6.12}$$

$$-PL_{km,\max} \leq f_{km} \leq PL_{km,\max} \tag{6.13}$$

where

k, m	Bus index
r_k	Curtailment of load k
γ_{km}	Susceptance of a line from bus k to bus m
$\mathbf{f}$	Power flow in vector form
$\mathbf{r}$	Load curtailment in vector form
$\mathbf{s}$	Bus-branch incidence matrix
$\mathbf{P_d}$	Bus load in vector form
$\mathbf{P_g}$	Bus power generation in vector form
$\mathbf{P_{g,min}}$, $\mathbf{P_{g,max}}$	Lower and upper generation limit in vector form

If $v_t^n > 0$, the corresponding *infeasibility cut* is generated as

$$v_t^n + \sum_{i=1}^{NG} \overline{\lambda}_{it}^n P_{Gi,\max}(I_{it} - \hat{I}_{it}^n) - \underline{\lambda}_{it}^n P_{Gi,\min}(I_{it} - \hat{I}_{it}^n) \leq 0 \tag{6.14}$$

The multiplier λ_{ibt}^n is interpreted as the marginal increase/decrease in unserved energy for a 1 MW increase in unit i power generation at hour t.

2. If $v_t^n = 0$, the *hourly optimal operation subproblem* is formulated as follows:

$$Min \quad w_t^n = \sum_{i=1}^{NG} c_{it} \, p_{it} \tag{6.15}$$

S.t.

First Kirchoff's law — bus power balance

$$\mathbf{sf} + \mathbf{P_g} = \mathbf{P_D} \tag{6.16}$$

Second Kirchoff's law – line power flow

$$f_{km} = \gamma_{km}(\theta_k - \theta_m) \tag{6.17}$$

$$\mathbf{P_g} \le \mathbf{P_{g,max}}\hat{\mathbf{I}} \qquad \overline{\boldsymbol{\pi}}$$
$$-\mathbf{P_g} \le -\mathbf{P_{g,min}}\hat{\mathbf{I}} \qquad \underline{\boldsymbol{\pi}} \tag{6.18}$$

$$-PL_{km,\max} \le f_{km} \le PL_{km,\max} \tag{6.19}$$

So the *feasibility cut* associated with the n[th] trial solution is

$$Z_{lower} \ge \sum_{t=1}^{T}\sum_{i=1}^{NG} st_i\alpha_{it} + sd_i\beta_{it} + \sum_t \left\{ w_t^n + \sum_{i=1}^{NG}\left[\overline{\pi}_{it}^n P_{Gi,\max}(I_{it} - \hat{I}_{it}^n) - \underline{\pi}_{it}^n P_{Gi,\min}(I_{it} - \hat{I}_{it}^n)\right]\right\} \tag{6.20}$$

The revised SCUC master problem (MP2) is given below which minimizes the operation cost subject to generation constraints as well as feasibility and infeasibility cuts.

$$Min \;\; Z_{lower}$$
$$Z_{lower} \ge \sum_{t=1}^{T}\sum_{i=1}^{NG} st_i\alpha_{it} + sd_i\beta_{it} \tag{6.21}$$

S.t.

Additional constraints (6.2)-(6.4).

Feasibility or infeasibility cuts which are given below:

If the optimal operation subproblems are feasible then the *feasibility cut* is

$$Z_{lower} \ge \sum_{t=1}^{T}\sum_{i=1}^{NG} st_i\alpha_{it} + sd_i\beta_{it} + \sum_t \left\{ w_t^n + \sum_{i=1}^{NG}\left[\overline{\pi}_{it}^n P_{Gi,\max}(I_{it} - \hat{I}_{it}^n) - \underline{\pi}_{it}^n P_{Gi,\min}(I_{it} - \hat{I}_{it}^n)\right]\right\} \tag{6.22}$$

If the optimal operation subproblem is infeasible then the *infeasibility cut* is:

$$v_t^n + \sum_{i=1}^{NG} \overline{\lambda}_{it}^n P_{Gi,\max}(I_{it} - \hat{I}_{it}^n) - \underline{\lambda}_{it}^n P_{Gi,\min}(I_{it} - \hat{I}_{it}^n) \le 0 \tag{6.23}$$

where n is the current number of iteration, and $\overline{\lambda}^n, \overline{\pi}^n, \underline{\lambda}^n, \underline{\pi}^n$ are multiplier vectors at the n^{th} iteration.

The important feature of the Benders decomposition is the availability of upper and lower bounds to the optimal solution at each iteration. These bounds can be used as an effective convergence criterion given as

$$2\left(Z_{upper} - Z_{lower}\right) \Big/ \left(Z_{upper} + Z_{lower}\right) \le \Delta \tag{6.24}$$

where $Z_{upper} = \sum_{t=1}^{T}\sum_{i=1}^{NG} c_i p_{it} + st_i\alpha_{it} + sd_i\beta_{it} = \sum_{t=1}^{T}\sum_{i=1}^{NG}(st_i\alpha_{it} + sd_i\beta_{it}) + \sum_{t=1}^{T} w_t$ (6.25)

Example 6.1

We use a three-bus system shown in Figure 6.2. The maximum energy not served requirement (ε) is 0 MW. Generator and line input data are given in Tables 6.2 and 6.3, respectively. Load data are shown in Table 6.4. The problem is defined as: The initial state of these two units is OFF. Minimum up/down time is one hour. Both reserve requirements and ramping constraints are ignored here. Calculate the optimal generation commitment of these two units.

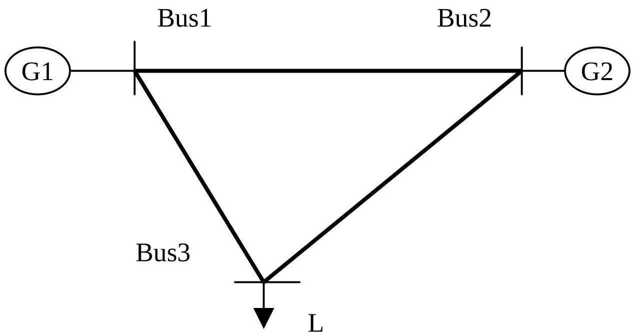

Figure 6.2 Three-Bus System Example

Table 6.2 Generator Data for 3-bus System

Unit	Min Capacity (MW)	Max Capacity (MW.)	Cost Coefficient ($/MW)	Startup Cost ($)	Shutdown Cost ($)
1	10	50	10	300	50
2	5	20	10	200	0

Table 6.3 Line Data for 3-bus System

Line	# of lines	Capacity/line (MW)
1-2	1	20
2-3	1	20
1-3	1	30

Table 6.4 Load Data

Hours	1	2
Load (MW)	35	45

The objective function of the original problem is

$$
\begin{aligned}
Min\ z = & 300*\alpha_{11} + 300*\alpha_{12} + 200*\alpha_{21} + 200*\alpha_{22} \\
& + 50*\beta_{11} + 50*\beta_{12} + 0*\beta_{21} + 0*\beta_{22} \\
& + 10*p_{11} + 10*p_{12} + 10*p_{21} + 10*p_{22}
\end{aligned}
$$

First, we solve the initial SCUC master problem.

<u>MP1: SCUC master problem iteration 1</u>

$$Min\ z_{lower}$$

$$
\begin{aligned}
z_{lower} = & 300*\alpha_{11} + 300*\alpha_{12} + 200*\alpha_{21} + 200*\alpha_{22} \\
& + 50*\beta_{11} + 50*\beta_{12} + 0*\beta_{21} + 0*\beta_{22}
\end{aligned}
$$

$$S.t. \quad \alpha_{11} - \beta_{11} = I_{11} - 0$$
$$\alpha_{12} - \beta_{12} = I_{12} - I_{11}$$
$$\alpha_{21} - \beta_{21} = I_{21} - 0$$
$$\alpha_{22} - \beta_{22} = I_{22} - I_{21}$$
$$50 * I_{11} + 20 * I_{21} \geq 35$$
$$50 * I_{12} + 20 * I_{22} \geq 45$$

Table 6.5 shows unit commitment solution $\mathbf{I}$ and the z_{lower} cost.

Table 6.5 Unit Commitment and operation cost at iteration 1

Hours	1	2	Z_{lower} ($)
Unit 1	1	1	300
Unit 2	0	0	

SP1: Feasibility check subproblem at hours 1-2 iteration 1

We check the feasibility of operation subproblem at hours 1-2 given the first trial of commitment.

The feasibility check at hour 1 is given as

$$Min \ r$$
$$S.t. - f_{12,1} - f_{13,1} + p_{11} = 0$$
$$- f_{23,1} + f_{12,1} + p_{21} = 0$$
$$f_{13,1} + f_{23,1} + r = 35$$
$$p_{11} \leq 50 * \hat{I}_{11} \qquad \lambda_1^u$$
$$- p_{11} \leq -10 * \hat{I}_{11} \qquad \lambda_1^l$$
$$p_{21} \leq 20 * \hat{I}_{21} \qquad \lambda_2^u$$
$$- p_{21} \leq -5 * \hat{I}_{21} \qquad \lambda_2^l$$
$$-20 \leq f_{12,1} \leq 20$$
$$-20 \leq f_{13,1} \leq 20$$
$$-30 \leq f_{23,1} \leq 30$$

The solution of feasibility check is $r = 0 \quad p_{11} = 35 \quad p_{21} = 0 \quad f_{12,1} = 15 \quad f_{13,1} = 20 \quad f_{23,1} = 15$.

The feasibility check subproblem at hour 2 of iteration 1 is given as

$$Min \ r$$
$$S.t. - f_{12,2} - f_{13,2} + p_{12} = 0$$
$$- f_{23,2} + f_{12,2} + p_{22} = 0$$
$$f_{13,2} + f_{23,2} + r = 45$$
$$p_{12} \leq 50 * \hat{I}_{12} \qquad \lambda_1^u$$
$$- p_{12} \leq -10 * \hat{I}_{12} \qquad \lambda_1^l$$
$$p_{22} \leq 20 * \hat{I}_{22} \qquad \lambda_2^u$$
$$- p_{22} \leq -5 * \hat{I}_{22} \qquad \lambda_2^l$$

$$-20 \le f_{12,2} \le 20$$
$$-20 \le f_{13,2} \le 20$$
$$-30 \le f_{23,2} \le 30$$

The feasibility check solution is $r = 5 \quad p_{12} = 40 \quad p_{22} = 0 \quad f_{12,2} = 20 \quad f_{13,2} = 20 \quad f_{23,2} = 20$. The dual multipliers of the operation sub-problem at hour 2 is $\lambda_1^u = 0 \quad \lambda_1^l = 0 \quad \lambda_2^u = -1 \quad \lambda_2^l = 0$.

The optimal operation subproblem at hour 2 is infeasible since $r = 5 \ge 0$. The infeasibility cut is as follows:

$$5 + 50 * \lambda_1^u * (I_{12} - \hat{I}_{12}) - 10 * \lambda_1^l * (I_{12} - \hat{I}_{12}) + 20 * \lambda_1^u * (I_{22} - \hat{I}_{22}) - 5 * \lambda_1^l * (I_{22} - \hat{I}_{22})$$
$$\Rightarrow$$
$$5 + 20 * (-1) * (I_{22} - 0) \le 0$$

MP2: SCUC master problem iteration 2

$$Min\ z_{lower}$$
$$z_{lower} \ge 300 * \alpha_{11} + 300 * \alpha_{12} + 200 * \alpha_{21} + 200 * \alpha_{22}$$
$$+ 50 * \beta_{11} + 50 * \beta_{12} + 0 * \beta_{21} + 0 * \beta_{22}$$
$$S.t. \quad \alpha_{11} - \beta_{11} = I_{11} - 0$$
$$\alpha_{12} - \beta_{12} = I_{12} - I_{11}$$
$$\alpha_{21} - \beta_{21} = I_{21} - 0$$
$$\alpha_{22} - \beta_{22} = I_{22} - I_{21}$$
$$50 * I_{11} + 20 * I_{21} \ge 35$$
$$50 * I_{12} + 20 * I_{22} \ge 45$$
$$5 + 20 * (-1) * (I_{22} - 0) \le 0$$

Table 6.6 shows the unit commitment solution at iteration 2.

Table 6.6 Unit Commitment and operation cost at iteration 2

Hours	**1**	**2**	**Z_{lower} ($)**
Unit 1	1	1	500
Unit 2	0	1	

SP2: Feasibility check subproblem at hours 1- 2 iteration 2

Check the feasibility at hours 1-2; here $r = 0$ which means the optimal operation subproblem is feasible at hours 1-2.

SP1: Optimal operation subproblem at hours 1- 2 iteration 2

The optimal operation subproblem at hour 1 of iteration 2 is given as

$$Min\ w_1 = 10 * p_{11} + 10 * p_{21}$$
$$S.t. - f_{12,1} - f_{13,1} + p_{11} = 0$$
$$- f_{23,1} + f_{12,1} + p_{21} = 0$$
$$f_{13,1} + f_{23,1} + r = 35$$
$$p_{11} \le 50 * \hat{I}_{11} \qquad \pi_1^u$$
$$- p_{11} \le -10 * \hat{I}_{11} \qquad \pi_1^l$$
$$p_{21} \le 20 * \hat{I}_{21} \qquad \pi_2^u$$
$$- p_{21} \le -5 * \hat{I}_{21} \qquad \pi_2^l$$
$$-20 \le f_{12,1} \le 20$$
$$-20 \le f_{13,1} \le 20$$
$$-30 \le f_{23,1} \le 30$$

The primal solution of feasibility check subproblem at hour 1 is $w_1 = 350 \quad p_{11} = 35 \quad p_{21} = 0$. The dual multipliers of the operation subproblem are $\pi_1^u = 0 \quad \pi_1^l = 0 \quad \pi_2^u = 0 \quad \pi_2^l = 0$.

The optimal operation subproblem at hour 2 of iteration 2 is given as

$$Min\ w_2 = 10 p_{12} + 10 p_{22}$$
$$S.t. - f_{12,2} - f_{13,2} + p_{12} = 0$$
$$- f_{23,2} + f_{12,2} + p_{22} = 0$$
$$f_{13,2} + f_{23,2} = 45$$
$$p_{12} \le 50 * \hat{I}_{12} \qquad \pi_1^u$$
$$- p_{12} \le -10 * \hat{I}_{12} \qquad \pi_1^l$$
$$p_{22} \le 20 * \hat{I}_{22} \qquad \pi_2^u$$
$$- p_{22} \le -5 * \hat{I}_{22} \qquad \pi_2^l$$
$$-20 \le f_{12,2} \le 20$$
$$-20 \le f_{13,2} \le 20$$
$$-30 \le f_{23,2} \le 30$$

The primal solution of feasibility check subproblem is $w_2 = 450 \quad p_{12} = 40 \quad p_{22} = 5$. The dual multipliers of the operation subproblem are $\pi_1^u = 0 \quad \pi_1^l = 0 \quad \pi_2^u = 0 \quad \pi_2^l = 0$.

We consider the feasibility cut for the third iteration because

$$z_{upper} = 500 + w_1 + w_2 = 500 + 350 + 450 = 1300 > z_{lower} = 500,$$
$$z_{lower} \ge 300 * \alpha_{11} + 300 * \alpha_{12} + 200 * \alpha_{21} + 200 * \alpha_{22}$$
$$+ 50 * \beta_{11} + 50 * \beta_{12} + 0 * \beta_{21} + 0 * \beta_{22}$$
$$+ 350 + 0$$
$$+ 450 + 0$$

MP2: SCUC master problem iteration 3:

$$Min\ z_{lower}$$

$$
\begin{aligned}
& z_{lower} \geq 300*\alpha_{11} + 300*\alpha_{12} + 200*\alpha_{21} + 200*\alpha_{22} \\
& + 50*\beta_{11} + 50*\beta_{12} + 0*\beta_{21} + 0*\beta_{22} \\
S.t.\quad & \alpha_{11} - \beta_{11} = I_{11} - 0 \\
& \alpha_{12} - \beta_{12} = I_{12} - I_{11} \\
& \alpha_{21} - \beta_{21} = I_{21} - 0 \\
& \alpha_{22} - \beta_{22} = I_{22} - I_{21} \\
& 50*I_{11} + 20*I_{21} \geq 35 \\
& 50*I_{12} + 20*I_{22} \geq 45 \\
& 5 + 20*(-1)*(I_{22} - 0) \leq 0 \\
& z_{lower} \geq 300*\alpha_{11} + 300*\alpha_{12} + 200*\alpha_{21} + 200*\alpha_{22} \\
& + 50*\beta_{11} + 50*\beta_{12} + 0*\beta_{21} + 0*\beta_{22} \\
& + 350 + 0 \\
& + 450 + 0
\end{aligned}
$$

Table 6.7 shows the unit commitment solution at iteration 3.

Table 6.7 Unit Commitment and operation cost at iteration 3

Hours	**1**	**2**	**Z_{lower} ($)**
Unit 1	1	1	1300
Unit 2	0	1	

It is obvious $z_{lower} = z_{upper} = 1300$ in next calculations and the final solution should be $z = 1300$.

7. Generation Resource Planning

The objective function of the generation resource planning is to minimize the investment and operation cost while satisfying the system reliability. The objective function is formulated as follows:

$$Min\ Y = \sum_{t}^{T} \sum_{i}^{CG} \left[CI_{it} * \left(X_{it} - X_{i(t-1)} \right) \right] + \sum_{t=1}^{T} \sum_{b=1}^{B} DT_{bt} * \sum_{i=1}^{NG} OC_{ibt} * P_{G,ibt} \tag{7.1}$$

where

i Existing or candidate unit index
b Load block index
t Planning year index
B Number of load blocks
CG Number of candidate units
T Planning horizon
NG Number of committed units
CI_{it} Capital investment for candidate unit i in year t
DT_{bt} Duration of load block b in year t
OC_{ibt} Operating cost unit i among committed units at load block b in year t

X_{it} State variable associated with candidate unit i in year t; 1: selected, 0: rejected. $(X_{i(t-1)} \le X_{it})$ $(X_{i0} = 0)$

$P_{G,ibt}$ Dispatched capacity of committed unit i at load block b in year t

The first terms of the objective function (7.1) is the construction cost for new generating units. The second item is the operation cost.

The set of planning constraints included in the resource planning problem include:
Constraints (7.2)-(7.5) represent the availability of capital investment funds in year t, projected resource capacity for year t, maximum number of units to be added at a planning year, and projected start of construction time, respectively.

$$\sum_{i=1}^{CG} CI_{it} * (X_{it} - X_{i(t-1)}) \le CI_t \qquad (t = 1,2,\cdots,T) \tag{7.2}$$

$$\sum_{i=1}^{CG} Cap_i * (X_{it} - X_{i(t-1)}) \le UC_t \qquad (t = 1,2,\cdots,T) \tag{7.3}$$

$$\sum_{i=1}^{CG} (X_{it} - X_{i(t-1)}) \le UN_t \qquad (t = 1,2,\cdots,T) \tag{7.4}$$

$$X_{it} = 0 \quad if \quad t < CT_i \qquad (i = 1,2,\cdots,PG)\ (t = 1,2,\cdots,T) \tag{7.5}$$

where

Cap_i Capacity of unit i

CI_t Capital investment in year t

CT_i Required construction time for candidate unit i

UC_t Upper limit for generating capacity added in year t

UN_t Upper limit for the # of units added in year t

Constraints (7.6) represent the system capacity requirement at planning year t. In other words, the total installed capacity of the candidate and existing units must meet the forecasted peak load demand and reserve capacity based on the system requirements.

$$\sum_{i=1}^{EG} Cap_i + \sum_{i=1}^{CG} Cap_i * X_{it} \ge P_{D,bt} + P_{R,bt} \quad (t = 1,2,\cdots,T)\ (b = peak\ load\ block) \tag{7.6}$$

where

EG Number of existing units

$P_{D,bt}$ Forecasted system load at load block b in year t

$P_{R,bt}$ Forecasted system reserve at load block b in year t

Additional constraints for representing a GENCO may also be included. For example, a GENCO applies constraint (7.7) for seeking the optimal location of a candidate unit among sites 1 through L:

$$\sum_{i \in CS} X_{it} \le 1 \qquad (t = T)\ (\mathrm{CS} = 1,\cdots,\mathrm{L}) \tag{7.7}$$

where

CS Set of candidate sites

Likewise, a GENCO may look for the best mix of new units for supplying the projected load. For instance, using constraint (7.8) the resource planner may consider two possible options for adding a 500

MW capacity. These options may include a 500 MW unit or five 100 MW units. The following constraint (7.8) is used to search the better option among possible alternatives (denoted by A and B alternatives in this case):

$$\begin{aligned} &X_{A1t} + X_{B1t} \le 1 \\ &X_{A1t} = X_{A2t} = \cdots = X_{Amt} \ (t = T) \\ &X_{B1t} = X_{B2t} = \cdots = X_{Bnt} \ (t = T) \\ &(A1, A2, \cdots, Am \in A\ Combination) \\ &(B1, B2, \cdots, Bn \in B\ Combination) \end{aligned} \tag{7.8}$$

System constraints (7.9)-(7.14) at load block b in planning year t are as follows:

The first Kirchoff's law — power node balance equations:

$$\mathbf{sf} + \mathbf{p} + \mathbf{r} = \mathbf{d} \tag{7.9}$$

The second Kirchoff's law for line flows

$$f_{mn} = (\delta_m - \delta_n) / x_{mn} \tag{7.10}$$

Generation limits for existing units,

$$P_{Gi,\min} \le P_{G,ibt} \le P_{Gi,\max} \tag{7.11}$$

Generation limits for candidate units,

$$P_{Gi,\min} * X_{it} \le P_{G,ibt} \le P_{Gi,\max} * X_{it} \tag{7.12}$$

Transmission flow limits:

$$-PL_{j,\max} \le f_{mn} \le PL_{j,\max} \qquad (j \in m, n) \tag{7.13}$$

Reliability requirement:

$$DT_{bt} \sum_{k=1}^{ND} r_{k,bt} \le \varepsilon_{bt} \tag{7.14}$$

where

j Transmission line index
k Load point index
m,n Bus index
ND Number of load points
$P_{Gi,min}$ Lower limit of generation of unit i
$P_{Gi,max}$ Upper limit of generation of unit i
$PL_{j,max}$ Capacity of line j from node m to n
r_k Curtailment of load k
f_{mn} Flow on line j from node m to node n
x_{mn} reactance of line j from node m to n
ε_{bt} Acceptable level of curtailment at load block b in year t
d Node load in vector form
f Power flow in vector form
p bus real generation in vector form
r Curtailment in vector form

s Node-branch incidence matrix

The Benders decomposition is used here in which the problem is decomposed into a master problem and two subproblems representing feasibility and optimal operation subproblems. The master, which is a mixed integer program (MIP), considers an investment plan for generating units based on the available types of units, suitable investment programs, and prospective locations based on the availability of site, and so on.

Once the candidate units are identified by the master problem, the feasibility subproblem will check whether this plan can meet system constraints (7.9)-(7.14). If the curtailment violations persist, the subproblem will form the corresponding Benders cut, which will be added to the master problem for solving the next iteration of the planning problem. Once the violations are removed, the solution of the optimal operation subproblem will measure the change in the total cost resulting from marginal changes in the proposed resource planning. The iterative solution will form one or more constraints for the next iteration of the optimal operation subproblem by using dual multipliers. The iterative process will continue until a converged optimal solution is found.

Solution Procedure

1. <u>The initial generation resource planning master problem (MP1)</u> is formulated as follow:

$$Min \;\; Z$$

$$Z \geq \sum_{t}^{T} \sum_{i}^{CG} \left[CI_{it} * \left(X_{it} - X_{i(t-1)} \right) \right] \tag{7.15}$$

Subject to constraints (7.2)-(7.8).

The initial plan must satisfy the reliability requirement (7.14) at load block b in planning year t to provide a secure supply while minimizing the cost of operation. <u>The nth operation subproblem SP1 (feasibility check)</u> is feasible if and only if the optimal value of the following subproblem is less than ε

$$Min \quad v^{n} = DT_{bt} \sum_{k=1}^{ND} r_{k,bt} \tag{7.16}$$

The objective (7.16) is to mitigate network violations and minimize the load curtailment by applying a generation redispatch. In this subproblem, we impose power balance (7.9), DC power flow equation (7.10), and generation and line flow limits (7.11-13). Note that the generation limits for candidate units can be rewritten as:

$$P_{G,ibt} \leq P_{Gi,\max} * X_{it} \qquad \overline{\lambda}_{ibt}^{n}$$

$$-P_{G,ibt} \leq -P_{Gi,\min} * X_{it} \qquad \underline{\lambda}_{ibt}^{n}$$

2. If constraint (7.14) is not satisfied, the corresponding <u>infeasibility cut</u> given by (7.17) will be generated as follows:

$$v^{n} + \sum_{i=1}^{CG} \overline{\lambda}_{ibt}^{n} P_{Gi,\max} (X_{it} - X_{it}^{n}) - \underline{\lambda}_{ibt}^{n} P_{Gi,\min} (X_{it} - X_{it}^{n}) \leq \varepsilon_{bt} \tag{7.17}$$

The multiplier λ_{ibt}^{n} is interpreted as the marginal decrease in unserved energy for a 1 MW generation increase in candidate unit i at load block b in the planning year t and associated with the nth trial plan. These n = 1,2,3,…,N-1 Benders cuts from the previous iterations are added to the master problem of resource planning to get the nth trial investment plan. The process will be repeated until a feasible plan is found for meeting the requirement (7.14) on system reliability.

3. If the above subproblem is feasible, then the optimal operation subproblem for every year and load block is formulated as follows:

$$Min\ w_{bt}^{n} = DT_{bt} * \sum_{i=1}^{NG} OC_{ibt} * P_{G,ibt} \tag{7.18}$$

Subject to the constraints (7.9)-(7.13). Similarly, note that the generation limits for candidate units can be rewritten as:

$$P_{G,ibt} \le P_{Gi,\max} * X_{it} \qquad \overline{\pi}_{ibt}^{n}$$

$$-P_{G,ibt} \le -P_{Gi,\min} * X_{it} \qquad \underline{\pi}_{ibt}^{n}$$

So the feasibility cut associated with the n^th trial solution is

$$\begin{aligned} Z \ge & \sum_{t} \sum_{i=1}^{CG} CI_{it} * (X_{it} - X_{i(t-1)}) \\ & + \sum_{t} \sum_{b} \left\{ w_{bt}^{n} + \left[\overline{\pi}_{ibt}^{n} P_{Gi,\max} (X_{it} - X_{it}^{n}) - \underline{\pi}_{ibt}^{n} P_{Gi,\min} (X_{it} - X_{it}^{n}) \right] \right\} \end{aligned} \tag{7.19}$$

The revised generation resource planning problem (MP2) (7.20) minimizes cost subject to planning constraints as well as feasibility and infeasibility cuts from the operation subproblems.

$$\begin{aligned} & Min\ \ Z \\ & \qquad Z \ge \sum_{t}^{T} \sum_{i}^{CG} \left[CI_{it} * \left(X_{it} - X_{i(t-1)} \right) \right] \end{aligned} \tag{7.20}$$

S.t.

Planning constraints (7.2)-(7.8).

Feasibility and infeasibility cuts from previous iterations

If all operation subproblems are feasible then the feasibility cut is:

$$\begin{aligned} Z \ge & \sum_{t} \sum_{i=1}^{CG} CI_{it} * (X_{it} - X_{i(t-1)}) \\ & + \sum_{t} \sum_{b} \left\{ w_{bt}^{n} + \left[\overline{\pi}_{ibt}^{n} P_{Gi,\max} (X_{it} - X_{it}^{n}) - \underline{\pi}_{ibt}^{n} P_{Gi,\min} (X_{it} - X_{it}^{n}) \right] \right\} \end{aligned} \tag{7.21}$$

If one or more operation subproblems (feasibility check) are infeasible then the infeasibility cuts are:

$$v^{n} + \sum_{i=1}^{CG} \overline{\lambda}_{ibt}^{n} P_{Gi,\max} (X_{it} - X_{it}^{n}) - \underline{\lambda}_{ibt}^{n} P_{Gi,\min} (X_{it} - X_{it}^{n}) \le \varepsilon_{bt} \tag{7.22}$$

where n is the current number of iterations

$\overline{\lambda}^{n}, \overline{\pi}^{n}, \underline{\lambda}^{n}, \underline{\pi}^{n}$ are the multiplier vectors at n^{th} iteration

The important feature of the Benders decomposition is the availability of upper and lower bounds to the optimal solution at each iteration. These bounds can be used as an effective convergence criterion. The convergence criterion is

$$\frac{2(Y-Z)}{(Y+Z)} \le \Delta \tag{7.23}$$

where

$$\begin{aligned} Y &= \sum_{t}^{T}\sum_{i}^{CG}\left[CI_{it} * \left(X_{it} - X_{i(t-1)}\right)\right] + \sum_{t=1}^{T}\sum_{b=1}^{B} DT_{bt} * \sum_{i=1}^{NG} OC_{ibt} * P_{G,ibt} \\ &= \sum_{t}^{T}\sum_{i}^{CG}\left[CI_{it} * \left(X_{it} - X_{i(t-1)}\right)\right] + \sum_{t=1}^{T}\sum_{b=1}^{B} w_{bt} \end{aligned} \tag{7.24}$$

Example

A 3-bus system, shown in Figure 1, is used to illustrate the proposed generation resource planning model. Existing and candidate generator, load and line data in per unit are given in Tables 2 through 5. We assume the studied planning period only has one-year interval. Loads are assumed constant during the period. Two candidate generators at bus 3 can be selected to supply the additional load at bus 2 in planning year 1. The maximum energy not served requirement (ε) is 0 p.u. in the planning year 1. Reserve requirements are not considered in this example.

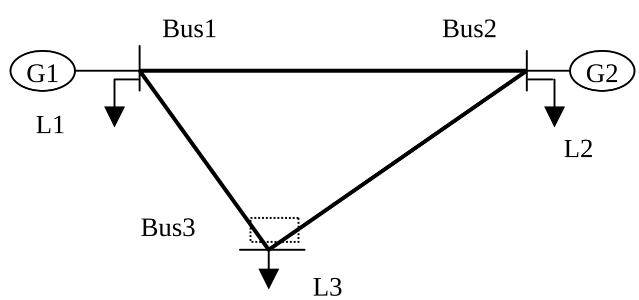

Figure 1 Three-Bus System Example

Table 2 Existing Generator Data for 3-bus System

Unit	Min Capacity (p.u.)	Max Capacity (p.u.)	Cost ($)/h
1	0.5	2.5	10 g_1
2	0.6	2.0	10 g_2

Table 3 Candidate Generator Data for 3-bus System

Unit	Min Capacity (p.u.)	Max Capacity (p.u.)	Cost ($)/h	Investment Cost/Unit ($)
3	0.6	3.0	8 g_3	50,000
4	0.6	3.0	10 g_4	40,000

Table 4 Load Data (MW)

Planning Year	L1	L2	L3
0	1	1	1
1	1	3	1

Table 5 Line Data for 3-bus System

Line	# of lines	Capacity/line (p.u.)
1-2	1	0.5
2-3	1	1.0
1-3	1	0.5

The original objective is

$$Min\ 50000*(x_3-0)+40000*(x_4-0)+8760*(10g_1+10g_2+8g_3+10g_4)$$

where x_3 represents the state of the candidate unit 3 and x_4 represents the state of the candidate unit 4.

First, we solve initial generation planning master problem.

<u>Generation planning master problem iteration 1:</u>

$$\begin{aligned} &Min\ z_{lower} \\ &S.t.\quad z_{lower} \geq 50000*(x_3-0)+40000*(x_4-0) \\ &\qquad 2.5+2.0+3.0x_3+3.0x_3 \geq 5.0 \\ &\qquad x_3, x_4 \in \{0,1\} \end{aligned}$$

The solution is $x_3=0, x_4=1$ and $z_{lower}=40000$.

<u>Operation subproblem iteration 1:</u>

We check the feasibility of operation subproblem given the first trial of generation planning schedule. The feasibility check is as follows:

$$\begin{aligned} &Min\ 8760*(r_1+r_2+r_3) \\ &S.t. -f_{12}-f_{13}+g_1+r_1=1 \\ &\quad -f_{23}+f_{12}+g_2+r_2=3 \\ &\quad f_{13}+f_{23}+g_3+g_4+r_3=1 \\ &\quad 0.5 \leq g_1 \leq 2.5 \\ &\quad 0.6 \leq g_2 \leq 2.0 \\ &\quad g_3 \leq 3.0*\hat{x}_3 \qquad \lambda_3^u \\ &\quad -g_3 \leq -0.6*\hat{x}_3 \qquad \lambda_3^l \\ &\quad g_4 \leq 3.0*\hat{x}_4 \qquad \lambda_4^u \\ &\quad -g_4 \leq -0.6*\hat{x}_4 \qquad \lambda_4^l \\ &\quad -0.5 \leq f_{12} \leq 0.5 \\ &\quad -0.5 \leq f_{13} \leq 0.5 \\ &\quad -1.0 \leq f_{23} \leq 1.0 \end{aligned}$$

The primal solution of feasibility check is $r = 0.0$. This means the trial schedule is feasible. The dual multipliers of the operation subproblem is:

$\lambda_3^u = 0 \quad \lambda_3^l = 0 \quad \lambda_4^u = 0 \quad \lambda_4^l = 0$

Operation subproblem iteration 1:

The feasible subproblem is as follows

$$\begin{aligned}
&Min\ w = 8760*(10*g_1 + 10*g_2 + 8*g_3 + 10g_4) \\
&S.t. - f_{12} - f_{13} + g_1 = 1 \\
&\quad - f_{23} + f_{12} + g_2 = 3 \\
&\quad f_{13} + f_{23} + g_3 + g_4 = 1 \\
&\quad 0.5 \le g_1 \le 2.5 \\
&\quad 0.6 \le g_2 \le 2.0 \\
&\quad g_3 \le 3.0*\hat{x}_3 \qquad \pi_3^u \\
&\quad -g_3 \le -0.6*\hat{x}_3 \qquad \pi_3^l \\
&\quad g_4 \le 3.0*\hat{x}_4 \qquad \pi_4^u \\
&\quad -g_4 \le -0.6*\hat{x}_4 \qquad \pi_4^l \\
&\quad -0.5 \le f_{12} \le 0.5 \\
&\quad -0.5 \le f_{13} \le 0.5 \\
&\quad -1.0 \le f_{23} \le 1.0
\end{aligned}$$

The primal solution of feasible subproblem is:

$w = 8760*50 = 438{,}000 \quad g_1 = 1.4839 \quad g_2 = 1.8037 \quad g_3 = 0.0 \quad g_4 = 1.7124$

$f_{12} = 0.4928 \quad f_{13} = -0.0139 \quad f_{23} = -0.6985$

The dual multipliers of the operation subproblem are:

$\pi_3^u = -(8760*2) = -17520 \quad \pi_3^l = 0 \quad \pi_4^u = 0 \quad \pi_4^l = 0$

Because $z_{upper} = 40000 + w = 40000 + 438{,}000 = 478{,}000 > z_{lower} = 40000$, the feasible cut for the second iteration is:

$$\begin{aligned}
z_{lower} &\ge 50000*(x_3 - 0) + 40000*(x_3 - 0) + 438{,}000 + 3.0*\pi_3^u*(x_3 - \hat{x}_3) - 0.6*\pi_3^l*(x_3 - \hat{x}_3) \\
&\quad + 3.0*\pi_4^u*(x_4 - \hat{x}_4) - 0.6*\pi_4^l*(x_4 - \hat{x}_4)
\end{aligned}$$

$\Rightarrow$

$$z_{lower} \ge 50000*(x_3 - 0) + 40000*(x_4 - 0) + 438{,}000 + 3.0*(-17520)*(x_3 - 0)$$

Generation planning master problem iteration 2:

$$\begin{aligned}
&Min\ z_{lower} \\
&S.t. \quad z_{lower} \ge 50000*(x_3 - 0) + 40000*(x_4 - 0) \\
&\qquad z_{lower} \ge 50000*(x_3 - 0) + 40000*(x_4 - 0) + 438{,}000 + 3.0*(-17520)*(x_3 - 0) \\
&\qquad 2.5 + 2.0 + 3.0x_3 + 3.0x_4 \ge 5.0 \\
&\qquad x_3, x_4 \in \{0, 1\}
\end{aligned}$$

The solution is:

$x_3 = 1, x_4 = 0$ and $z_{lower} = 435{,}440$.

Operation subproblem iteration 2:

Because of $r = 0.0$ based on the above investment strategy, the trial schedule is feasible. The dual multipliers of the operation subproblem is:

$\lambda_3^u = 0 \quad \lambda_3^l = 0 \quad \lambda_4^u = 0 \quad \lambda_4^l = 0$

Operation subproblem iteration 2:

The feasible subproblem is as follows

$$
\begin{aligned}
&Min\ w = 8760*(10*g_1 + 10*g_2 + 8*g_3 + 10g_4) \\
&S.t. - f_{12} - f_{13} + g_1 = 1 \\
&\quad - f_{23} + f_{12} + g_2 = 3 \\
&\quad f_{13} + f_{23} + g_3 + g_4 = 1 \\
&\quad 0.5 \le g_1 \le 2.5 \\
&\quad 0.6 \le g_2 \le 2.0 \\
&\quad g_3 \le 3.0*\hat{x}_3 \qquad \pi_3^u \\
&\quad -g_3 \le -0.6*\hat{x}_3 \qquad \pi_3^l \\
&\quad g_4 \le 3.0*\hat{x}_4 \qquad \pi_4^u \\
&\quad -g_4 \le -0.6*\hat{x}_4 \qquad \pi_4^l \\
&\quad -0.5 \le f_{12} \le 0.5 \\
&\quad -0.5 \le f_{13} \le 0.5 \\
&\quad -1.0 \le f_{23} \le 1.0
\end{aligned}
$$

The primal solution of feasible subproblem is:

$w = 394{,}200 \quad g_1 = 0.9722 \quad g_2 = 1.5278 \quad g_3 = 2.5 \quad g_4 = 0.0$

$f_{12} = 0.4722 \quad f_{13} = -0.5 \quad f_{23} = -1.0$

The dual multipliers of the operation subproblem is:

$\pi_3^u = 0 \quad \pi_3^l = 0 \quad \pi_4^u = 0 \quad \pi_4^l = -(8760*2) = -17520$

Because $z_{upper} = 50000 + w = 50000 + 394{,}200 = 444{,}200 > z_{lower} = 435{,}440$, the feasible cut for the second iteration is:

$$
\begin{aligned}
z_{lower} \ge\ & 50000*(x_3 - 0) + 40000*(x_4 - 0) + 394{,}200 + 3.0*\pi_3^u*(x_3 - \hat{x}_3) - 0.6*\pi_3^l*(x_3 - \hat{x}_3) \\
& + 3.0*\pi_4^u*(x_4 - \hat{x}_4) - 0.6*\pi_4^l*(x_4 - \hat{x}_4)
\end{aligned}
$$

$\Rightarrow$

$$z_{lower} \ge 50000*(x_3 - 0) + 40000*(x_4 - 0) + 394{,}200 - 0.6*(-17520)*(x_4 - 0)$$

Generation planning master problem iteration 3:

$$
\begin{aligned}
& Min\ z_{lower} \\
& S.t. \quad z_{lower} \geq 50000*(x_3-0)+40000*(x_4-0) \\
& \qquad z_{lower} \geq 50000*(x_3-0)+40000*(x_4-0)+438{,}000+3.0*(-17520)*(x_3-0) \\
& \qquad z_{lower} \geq 50000*(x_3-0)+40000*(x_4-0)+394{,}200-0.6*(-17520)*(x_4-0) \\
& \qquad 2.5+2.0+3.0x_3+3.0x_4 \geq 5.0 \\
& \qquad x_3, x_4 \in \{0,1\}
\end{aligned}
$$

The solution is:

$x_3 = 1, x_4 = 0$ and $z_{lower} = 444{,}200$.

Operation subproblem iteration 3:

Because $r = 0.0$ based on the above investment strategy, the trial schedule is feasible.

Operation subproblem iteration 3:

According to calculations, the primal solution of feasible subproblem is:

$w = 394{,}200 \quad g_1 = 0.9722 \quad g_2 = 1.5278 \quad g_3 = 2.5 \quad g_4 = 0.0$

$f_{12} = 0.4722 \quad f_{13} = -0.5 \quad f_{23} = -1.0$

Because $z_{upper} = 50000 + w = 50000 + 394{,}200 = z_{lower} = 444{,}200$, the optimal solution is obtained, which show that selecting the economical unit 3 can save more money, though it has a higher investment cost than the candidate unit 4.

8. TRANSMISSION PLANNING

The objective function of the transmission planning is to minimize the investment and operation cost under steady state while satisfying the system reliability requirement for each scenario φ. The objective function is formulated as follows:

$$Min\ Y = \sum_{t=1}^{T} \sum_{j=1}^{CL} \left[CI_{jt} * \left(X_{jt} - X_{j(t-1)} \right) \right] + \sum_{t=1}^{T} \sum_{b=1}^{B} DT_{bt} * \sum_{i=1}^{NG} OC_{ibt} * P_{G,ibt} \qquad (8.1)$$

where

- i Unit index
- j Candidate line index
- b Load block index
- t Planning year index
- B Number of load blocks
- CL Number of candidate lines
- T Planning horizon
- NG Number of committed units
- CI_{jt} Capital investment for candidate line j in year t
- DT_{bt} Duration of load block b in year t
- OC_{ibt} Operating cost unit i among committed units at load block b in year t
- X_{jt} State variable associated with candidate line j in year t; 1: selected, 0: rejected. $(X_{i(t-1)} \le X_{it})$ $(X_{i0} = 0)$.
- $P_{G,ibt}$ Dispatched capacity of committed unit i at load block b in year t

The first terms in the objective function (8.1) is the construction cost for new transmission lines. The second item is the operation cost.

The set of planning constraints included in the transmission planning problem are:
Constraints (8.2)-(8.5) represent the availability of capital investment funds in year t, projected line capacity for year t, maximum number of lines to be added at a planning year, and projected construction time, respectively.

$$\sum_{j=1}^{CL} CI_{jt} * (X_{jt} - X_{j(t-1)}) \le CI_t \qquad (t = 1,2,\cdots,T) \qquad (8.2)$$

$$\sum_{j=1}^{CL} Cap_j * (X_{jt} - X_{j(t-1)}) \le UC_t \qquad (t = 1,2,\cdots,T) \qquad (8.3)$$

$$\sum_{j=1}^{CL} (X_{jt} - X_{j(t-1)}) \le UN_t \qquad (t = 1,2,\cdots,T) \qquad (8.4)$$

$$X_{jt} = 0 \quad if \quad t < CT_j \qquad (j = 1,2,\cdots,NL)\ (t = 1,2,\cdots,T) \qquad (8.5)$$

where

- Cap_j Capacity of line j
- CI_t Capital investment in year t
- CT_j Required construction time for candidate line j
- UC_t Upper limit for line capacity added in year t
- UN_t Upper limit for the # of lines added in year t

Additional constraints for representing a TRANSCO may also be included. For example, a TRANSCO applies constraint (8.6) for seeking the optimal location of a candidate line among corridors 1 through L:

$$\sum_{j \in CS} X_{jt} \le 1 \qquad (t = T)\ (\mathrm{CS} = 1, \cdots, \mathrm{L}) \tag{8.6}$$

where

CS Set of candidate line corridors

Likewise, a TRANSCO may look for the best mix of new lines for transferring the electricity to the projected load. For instance, using constraint (8.7) the transmission planner may consider two possible options for adding a 200 MW capacity. These options may include a 200 MW line or two 100 MW lines. The following constraint (8.7,8.8) is used to search the better option among possible alternatives (denoted by A and B alternatives in this case):

$$X_{A1t} + X_{B1t} \le 1$$
$$X_{A1t} = X_{A2t} = \cdots = X_{Amt}\ (t = T)$$
$$X_{B1t} = X_{B2t} = \cdots = X_{Bnt}\ (t = T) \qquad (8.7, 8.8)$$
$$(A1, A2, \cdots, Am \in A\ Combination)$$
$$(B1, B2, \cdots, Bn \in B\ Combination)$$

System constraints (8.9)-(8.15) for each scenario φ at load block b in planning year t are as follows:

The first Kirchoff's law — power node balance equations:

$$\mathbf{sf} + \mathbf{p} + \mathbf{r} = \mathbf{d} \quad (\varphi) \tag{8.9}$$

The second Kirchoff's law for existing lines

$$f_{mn} - \gamma_{mn}(\theta_m - \theta_n) = 0 \qquad (\varphi) \tag{8.10}$$

The second Kirchoff's law for candidate lines

$$\left| f_{mn} - \gamma_{mn}(\theta_m - \theta_n) \right| \le M_j * (1 - X_{jt}) \quad (j \in m, n) \quad (\varphi) \tag{8.11}$$

where M is a large positive number.

Transmission flow limits for existing lines:

$$-PL_{j,\max} \le f_{mn} \le PL_{j,\max} \qquad (j \in m, n) \qquad (\varphi) \tag{8.12}$$

Transmission flow limits for candidate lines:

$$\left| f_{mn} \right| \le PL_{j,\max} * X_{jt} \qquad (j \in m, n) \quad (\varphi) \tag{8.13}$$

Generation limits:

$$P_{Gi,\min} \le P_{G,ibt} \le P_{Gi,\max} \qquad (\varphi) \tag{8.14}$$

Reliability requirement:

$$DT_{bt} * \sum_{k=1}^{ND} r_{k,bt} \le \varepsilon_{bt} \qquad (\varphi) \tag{8.15}$$

where

k Load point index

m,n Bus index

ND Number of load points

$P_{Gi,min}$ Lower limit of generation of unit i

$P_{Gi,max}$ Upper limit of generation of unit i

$PL_{mn,max}$ Capacity of line from node m to node n

r_k Curtailment of load k

f_{mn} Flow on line j from node m to node n

γ_{mn} Line susceptance in vector form

ε_{bt} Acceptable level of curtailment at load block b in year t

φ Index of scenario (including the steady state and contingencies)

d Node load in vector form

f Power flow in vector form

p Bus real generation in vector form

r Curtailment in vector form

s Node-branch incidence matrix

The Benders decomposition is used here in which the problem is decomposed into a master problem and two subproblems representing feasibility and optimal operation subproblems. The master, which is a mixed integer program (MIP), considers an investment plan for transmission lines based on the available types of lines, suitable investment programs, and prospective locations based on the availability of corridor, and so on.

Once the candidate lines are identified by the master problem, the feasibility subproblem will check whether this plan can meet system constraints (8.9)-(8.15). If the curtailment violations persist, the subproblem will form the corresponding Benders cut, which will be added to the master problem for solving the next iteration of the planning problem. Once the violations are removed, the solution of the optimal operation subproblem will measure the change in the total cost resulting from marginal changes in the proposed transmission planning. The iterative solution will form one or more constraints for the next iteration of the optimal operation subproblem by using dual multipliers. The iterative process will continue until a converged optimal solution is found.

Solution Procedure

The initial transmission planning master problem is formulated as follow:

$Min \;\; Z$

$$Z \geq \sum_{t}^{T} \sum_{j}^{CL} \left[CI_{jt} * \left(X_{jt} - X_{j(t-1)} \right) \right] \tag{8.16}$$

Subject to constraints (8.2)-(8.8).

The initial plan must satisfy the reliability requirement (8.15) for each scenario φ at load block b in planning year t to provide a secure supply while minimizing the cost of operation. The n^{th} operation subproblem is feasible if and only if the optimal value of the following feasibility check subproblem is less than ε

$$Min \quad v_t^n = DT_{bt} * \sum_{k=1}^{ND} r_{k,bt} \qquad (\varphi) \tag{8.17}$$

The objective (8.17) is to mitigate network violations and minimize the load curtailment by applying a generation redispatch. In this subproblem, constraints (8.9-8.14) are taken into account. Note that the constraints (8.11) and (8.13) corresponding to candidate lines can be rewritten as

$$f_{mn} - \gamma_{mn}(\theta_m - \theta_n) \le M_j * (1 - X_{jt}) \quad (j \in m,n) \quad (\varphi) \qquad \overline{\pi}$$

$$-(f_{mn} - \gamma_{mn}(\theta_m - \theta_n)) \le M_j * (1 - X_{jt}) \quad (j \in m,n) \quad (\varphi) \qquad \underline{\pi}$$

$$f_{mn} \le PL_{j,\max} * X_{jt} \quad (j \in m,n) \quad (\varphi) \qquad \overline{\lambda}$$

$$-f_{mn} \le PL_{j,\max} * X_{jt} \quad (j \in m,n) \quad (\varphi) \qquad \underline{\lambda}$$

If constraint (8.15) is not satisfied, the corresponding infeasibility cut given by (8.18) will be generated as follows:

$$v_t^n + \sum_{j=1}^{CL} (\overline{\lambda}_{ibt}^n + \underline{\lambda}_{ibt}^n) PL_{j,\max} (X_{jt} - X_{jt}^n) - (\overline{\pi}_{jbt}^n + \underline{\pi}_{jbt}^n) M_j (X_{jt} - X_{jt}^n) \le \varepsilon_{bt} \tag{8.18}$$

These n = 1,2,3,…,N-1 Benders cuts from the previous iterations are added to the master problem of transmission planning to get the n^{th} trial investment plan. The process will be repeated until a feasible plan is found for meeting the requirement (8.15) on system reliability.

If the above subproblem is feasible, then the optimal operation subproblem under the steady state for every year and load block is formulated as follows:

$$Min \ w_{bt}^n = DT_{bt} * \sum_{i=1}^{NG} OC_{ibt} * P_{G,ibt} \tag{8.19}$$

Subject to the constraints (8.9)-(8.14). Note that the constraints (8.11) and (8.13) can be rewritten as

$$f_{mn} - \gamma_{mn}(\theta_m - \theta_n) \le M_j * (1 - X_{jt}) \quad (j \in m,n) \qquad \overline{\beta}$$

$$-(f_{mn} - \gamma_{mn}(\theta_m - \theta_n)) \le M_j * (1 - X_{jt}) \quad (j \in m,n) \qquad \underline{\beta}$$

$$f_{mn} \le PL_{j,\max} * X_{jt} \quad (j \in m,n) \qquad \overline{\alpha}$$

$$-f_{mn} \le PL_{j,\max} * X_{jt} \quad (j \in m,n) \qquad \underline{\alpha}$$

Therefore, the feasibility cut associated with the n^{th} trial solution is

$$Z \ge \sum_t \sum_{j=1}^{CL} CI_{jt} * (X_{jt} - X_{j(t-1)})$$
$$+ \sum_t \sum_b \left\{ w_{bt}^n + \sum_{j=1}^{CL} (\overline{\alpha}_{ibt}^n + \underline{\alpha}_{ibt}^n) PL_{j,\max} (X_{jt} - X_{jt}^n) - (\overline{\beta}_{jbt}^n + \underline{\beta}_{jbt}^n) M_j (X_{jt} - X_{jt}^n) \right\} \tag{8.20}$$

Thus, the revised transmission planning problem MP1 (8.21) minimizes cost subject to planning constraints as well as feasibility and infeasibility cuts from the operation subproblems.

$$Min \ \ Z$$

$$Z \ge \sum_t^T \sum_j^{CL} \left[CI_{jt} * \left(X_{jt} - X_{j(t-1)} \right) \right] \tag{8.21}$$

S.t. Planning constraints (8.2)-(8.7).

Feasibility and infeasibility cuts from previous iterations are:

If all operation subproblems are feasible then the feasibility cut is:

$$Z \geq \sum_{t} \sum_{j=1}^{CL} CI_{jt} * (X_{jt} - X_{j(t-1)})$$
$$+ \sum_{t} \sum_{b} \left\{ w_{bt}^{n} + \sum_{j=1}^{CL} (\overline{\alpha}_{ibt}^{n} + \underline{\alpha}_{ibt}^{n}) PL_{j,\max} (X_{jt} - X_{jt}^{n}) - (\overline{\beta}_{jbt}^{n} + \underline{\beta}_{jbt}^{n}) M_{j} (X_{jt} - X_{jt}^{n}) \right\} \tag{8.22}$$

However, if one or more operation subproblem (feasibility check) is infeasible then the infeasibility cuts are:

$$v_{t}^{n} + \sum_{j=1}^{CL} (\overline{\lambda}_{ibt}^{n} + \underline{\lambda}_{ibt}^{n}) PL_{j,\max} (X_{jt} - X_{jt}^{n}) - (\overline{\pi}_{jbt}^{n} + \underline{\pi}_{jbt}^{n}) M_{j} (X_{jt} - X_{jt}^{n}) \leq \varepsilon_{bt} \tag{8.23}$$

where n is the current number of iterations

$\overline{\lambda}^{n}, \underline{\lambda}^{n}, \overline{\pi}^{n}, \underline{\pi}^{n}$ are the multiplier vectors at n^{th} iteration

The important feature of the Benders decomposition is the availability of upper (Y) and lower bounds (Z) to the optimal solution at each iteration. These bounds can be used as an effective convergence criterion. The convergence criterion is

$$\frac{2(Y - Z)}{(Y + Z)} \leq \Delta \tag{8.24}$$

where

$$Y = \sum_{t}^{T} \sum_{j}^{CL} \left[CI_{jt} * \left(X_{jt} - X_{j(t-1)} \right) \right] + \sum_{t=1}^{T} \sum_{b=1}^{B} DT_{bt} * \sum_{i=1}^{NG} OC_{ibt} * P_{G,ibt}$$
$$= \sum_{t}^{T} \sum_{j}^{CL} \left[CI_{jt} * \left(X_{jt} - X_{j(t-1)} \right) \right] + \sum_{t=1}^{T} \sum_{b=1}^{B} w_{bt} \tag{8.25}$$

The solution framework is shown as follows:

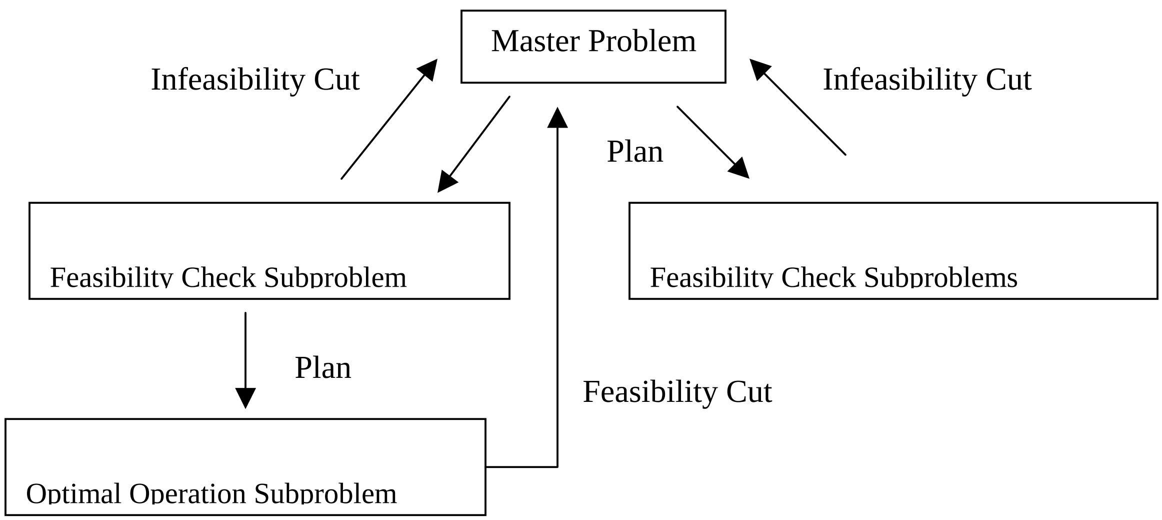

Figure 1 Solution Framework

Example 1

A 4-bus system, shown in Figure 2, is used to illustrate the proposed transmission planning model. Bus 1 is the slack bus. Existing and candidate line, generator, load data are given in Tables 1 through 3. We assume the studied planning period only has one-year interval. Loads are assumed constant during the period. At least one of the two candidate lines should be built to transfer the electricity to the new load bus in the planning year 1. The maximum energy not served requirement (ε) is 0 p.u. in the planning year 1.

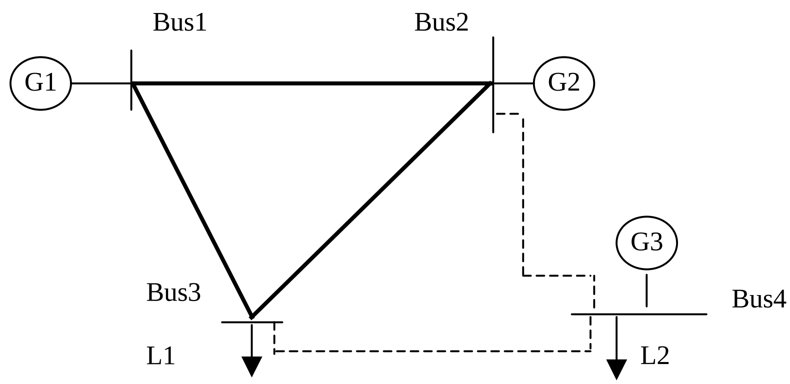

Figure 2 Three-Bus System Example

Table 1 Existing and candidate line Data for the 4-bus System

Line	From bus	To bus	Reactance (pu)	Capacity (MW)	Investment cost ($)
1	2	4	0.2	100	6,000,000
2	3	4	0.2	100	5,000,000
3	1	2	0.1	150	-
4	2	3	0.2	100	-
5	1	3	0.1	150	-

Table 2 Generator Data for the 4-bus System

Unit	Min Capacity (MW)	Max Capacity (MW)	Cost ($/MWh)
1	50	150	10
2	100	200	8
3	50	100	10

Table 3 Load Data (MW)

Planning Year	L1	L2
1	200	200

The original objective is $Min\ 6{,}000{,}000*(x_1-0)+5{,}000{,}000*(x_2-0)+8760*(10g_1+8g_2+10g_3)$

where x_1 represents the state of candidate line 1 extending from 2-4 and x_2 represents the state of candidate line 2 extending from 3-4.

First, we solve the initial transmission planning master problem.

<u>Transmission planning master problem iteration 1:</u>

$Min\ z_{lower}$

$S.t.\quad z_{lower} \geq 6{,}000{,}000*(x_1-0)+5{,}000{,}000*(x_2-0)$

$x_1+x_2 \geq 1$

$x_1, x_2 \in \{0, 1\}$

The solution is: $x_1=0, x_2=1$ and $z_{lower}=5{,}000{,}000$.

<u>Operation subproblem iteration 1:</u>

We check the feasibility of operation subproblem given the first trial of transmission planning schedule. Assume M= 1000. The feasibility check is as follows:

$$
\begin{aligned}
&Min\ 8760*(r_1+r_2)\\
&S.t. - f_{12} - f_{13} + g_1 = 0\\
&\quad - f_{23} + f_{12} - f_{24} + g_2 = 0\\
&\quad f_{13} + f_{23} - f_{34} + r_1 = 200\\
&\quad f_{24} + f_{34} + g_3 + r_2 = 200\\
&\quad f_{12} - (\theta_1-\theta_2)/0.1 = 0\\
&\quad f_{13} - (\theta_1-\theta_3)/0.1 = 0\\
&\quad f_{23} - (\theta_2-\theta_3)/0.2 = 0\\
&\quad f_{24} - (\theta_2-\theta_4)/0.2 \le 1000*(1-\hat{x}_1) \qquad \overline{\pi}_1\\
&\quad -(f_{24} - (\theta_2-\theta_4)/0.2) \le 1000*(1-\hat{x}_1) \qquad \underline{\pi}_1\\
&\quad f_{34} - (\theta_3-\theta_4)/0.2 \le 1000*(1-\hat{x}_2) \qquad \overline{\pi}_2\\
&\quad -(f_{34} - (\theta_3-\theta_4)/0.2) \le 1000*(1-\hat{x}_2) \qquad \underline{\pi}_2\\
&\quad -150 \le f_{12} \le 150\\
&\quad -150 \le f_{13} \le 150\\
&\quad -100 \le f_{23} \le 100\\
&\quad f_{24} \le 100*\hat{x}_1 \qquad \overline{\lambda}_1\\
&\quad -f_{24} \le 100*\hat{x}_1 \qquad \underline{\lambda}_1\\
&\quad f_{34} \le 100*\hat{x}_2 \qquad \overline{\lambda}_2\\
&\quad -f_{34} \le 100*\hat{x}_2 \qquad \underline{\lambda}_2\\
&\quad 50 \le g_1 \le 150\\
&\quad 100 \le g_2 \le 200\\
&\quad 50 \le g_3 \le 100\\
&\quad \theta_1 = 0
\end{aligned}
$$

The primal solutions of feasibility check are

$f_{12} = -50,\ f_{13} = 150,\ f_{23} = 100,\ f_{24} = 0,\ f_{34} = 68.6578\ \theta_1 = 0, \theta_2 = 5, \theta_3 = -15, \theta_4 = -28.7316$

$g_1 = 100, g_2 = 150, g_3 = 100$ and $r = r_1 + r_2 = 18.6578 + 31.3422 = 50.0$, which means the trial schedule is infeasible. The dual multipliers of the operation subproblem are:

$\overline{\lambda}_1 = 0 \quad \underline{\lambda}_1 = 8760*(-1) \quad \overline{\lambda}_2 = 0 \quad \underline{\lambda}_2 = 0$

$\overline{\pi}_1 = 0 \quad \underline{\pi}_1 = 0 \quad \overline{\pi}_2 = 0 \quad \underline{\pi}_2 = 0$

Therefore, the infeasibility cut is given as

$$
\begin{aligned}
&8760*r + (\overline{\lambda}_1 + \underline{\lambda}_1)*100*(x_1-\hat{x}_1) + (\overline{\lambda}_2 + \underline{\lambda}_2)*100*(x_2-\hat{x}_2)\\
&\qquad -(\overline{\pi}_1 + \underline{\pi}_1)*1000*(x_1-\hat{x}_1) - (\overline{\pi}_2 + \underline{\pi}_2)*1000*(x_2-\hat{x}_2) \le 0\\
&\Rightarrow\\
&438{,}000 - 876{,}000*(x_1 - 0) \le 0
\end{aligned}
$$

Transmission planning master problem iteration 2:

$$\begin{aligned}
&Min\ z_{lower}\\
&S.t.\quad z_{lower} \geq 6{,}000{,}000*(x_1-0)+5{,}000{,}000*(x_2-0)\\
&\qquad x_1+x_2 \geq 1\\
&\qquad 438{,}000-876{,}000*(x_1-0)\leq 0\\
&\qquad x_1, x_2 \in \{0,1\}
\end{aligned}$$

The solution is: $x_1=1, x_2=0$ and $z_{lower}=6{,}000{,}000$.

Operation subproblem iteration 2:
The feasibility check is as follows:

$$\begin{aligned}
&Min\ 8760*(r_1+r_2)\\
&S.t. -f_{12}-f_{13}+g_1=0\\
&\quad -f_{23}+f_{12}-f_{24}+g_2=0\\
&\quad f_{13}+f_{23}-f_{34}+r_1=200\\
&\quad f_{24}+f_{34}+g_3+r_2=200\\
&\quad f_{12}-(\theta_1-\theta_2)/0.1 = 0\\
&\quad f_{13}-(\theta_1-\theta_3)/0.1 = 0\\
&\quad f_{23}-(\theta_2-\theta_3)/0.2 = 0\\
&\quad f_{24}-(\theta_2-\theta_4)/0.2 \leq 1000*(1-\hat{x}_1) \qquad \overline{\pi}_1\\
&\quad -(f_{24}-(\theta_2-\theta_4)/0.2)\leq 1000*(1-\hat{x}_1) \qquad \underline{\pi}_1\\
&\quad f_{34}-(\theta_3-\theta_4)/0.2 \leq 1000*(1-\hat{x}_2) \qquad \overline{\pi}_2\\
&\quad -(f_{34}-(\theta_3-\theta_4)/0.2)\leq 1000*(1-\hat{x}_2) \qquad \underline{\pi}_2\\
&\quad -150\leq f_{12}\leq 150\\
&\quad -150\leq f_{13}\leq 150\\
&\quad -100\leq f_{23}\leq 100\\
&\quad f_{24}\leq 100*\hat{x}_1 \qquad \overline{\lambda}_1\\
&\quad -f_{24}\leq 100*\hat{x}_1 \qquad \underline{\lambda}_1\\
&\quad f_{34}\leq 100*\hat{x}_2 \qquad \overline{\lambda}_2\\
&\quad -f_{34}\leq 100*\hat{x}_2 \qquad \underline{\lambda}_2\\
&\quad 50\leq g_1\leq 150\\
&\quad 100\leq g_2\leq 200\\
&\quad 50\leq g_3\leq 100\\
&\quad \theta_1=0
\end{aligned}$$

Because $r=0.0$ based on the above investment strategy, the trial schedule is feasible.

Operation subproblem iteration 2:

The feasible subproblem is as follows

$$
\begin{aligned}
&Min\ 8760*(10*g_1+8*g_2+10*g_3)\\
&S.t. -f_{12}-f_{13}+g_1=0\\
&\quad -f_{23}+f_{12}-f_{24}+g_2=0\\
&\quad f_{13}+f_{23}-f_{34}=200\\
&\quad f_{24}+f_{34}+g_3=200\\
&\quad f_{12}-(\theta_1-\theta_2)/0.1=0\\
&\quad f_{13}-(\theta_1-\theta_3)/0.1=0\\
&\quad f_{23}-(\theta_2-\theta_3)/0.2=0\\
&\quad f_{24}-(\theta_2-\theta_4)/0.2\le 1000*(1-\hat{x}_1) && \overline{\beta}_1\\
&\quad -(f_{24}-(\theta_2-\theta_4)/0.2)\le 1000*(1-\hat{x}_1) && \underline{\beta}_1\\
&\quad f_{34}-(\theta_3-\theta_4)/0.2\le 1000*(1-\hat{x}_2) && \overline{\beta}_2\\
&\quad -(f_{34}-(\theta_3-\theta_4)/0.2)\le 1000*(1-\hat{x}_2) && \underline{\beta}_2\\
&\quad -150\le f_{12}\le 150\\
&\quad -150\le f_{13}\le 150\\
&\quad -100\le f_{23}\le 100\\
&\quad f_{24}\le 100*\hat{x}_1 && \overline{\alpha}_1\\
&\quad -f_{24}\le 100*\hat{x}_1 && \underline{\alpha}_1\\
&\quad f_{34}\le 100*\hat{x}_2 && \overline{\alpha}_2\\
&\quad -f_{34}\le 100*\hat{x}_2 && \underline{\alpha}_2\\
&\quad 50\le g_1\le 150\\
&\quad 100\le g_2\le 200\\
&\quad 50\le g_3\le 100\\
&\quad \theta_1=0
\end{aligned}
$$

The primal solutions of feasible subproblem are:

$w=31{,}536{,}000$

$f_{12}=-25,\ f_{13}=125,\ f_{23}=75,\ f_{24}=100,\ f_{34}=0\ \ \theta_1=0,\ \theta_2=2.5,\ \theta_3=-12.5,\ \theta_4=-17.5$

$g_1=100,\ g_2=200,\ g_3=100$

The dual multipliers of the operation subproblem is:

$\overline{\alpha}_1=0\quad \underline{\alpha}_1=0\quad \overline{\alpha}_2=0\quad \underline{\alpha}_2=0$

$\overline{\beta}_1=0\quad \underline{\beta}_1=0\quad \overline{\beta}_2=0\quad \underline{\beta}_2=0$

Because $z_{upper}=6{,}000{,}000+w=6{,}000{,}000+31{,}536{,}000=37{,}536{,}000>z_{lower}=6{,}000{,}000$, the feasible cut for the second iteration is:

$$
\begin{aligned}
z_{lower}\ge\ & 6{,}000{,}000*(x_1-0)+5{,}000{,}000*(x_2-0)+31{,}536{,}000\\
&+(\overline{\alpha}_1+\underline{\alpha}_1)*100*(x_1-\hat{x}_1)+(\overline{\alpha}_2+\underline{\alpha}_2)*100*(x_2-\hat{x}_2)\\
&-(\overline{\beta}_1+\underline{\beta}_1)*1000*(x_1-\hat{x}_1)-(\overline{\beta}_2+\underline{\beta}_2)*1000*(x_2-\hat{x}_2)
\end{aligned}
$$

$\Rightarrow$

$z_{lower}\ge 6{,}000{,}000*(x_1-0)+5{,}000{,}000*(x_2-0)+31{,}536{,}000$

Transmission planning master problem iteration 3:

$$
\begin{aligned}
& Min\ z_{lower} \\
& S.t. \quad z_{lower} \geq 6{,}000{,}000*(x_1-0)+5{,}000{,}000*(x_2-0) \\
& \qquad x_1+x_2 \geq 1 \\
& \qquad 438{,}000-876{,}000*(x_1-0) \leq 0 \\
& \qquad z_{lower} \geq 6{,}000{,}000*(x_1-0)+5{,}000{,}000*(x_2-0)+31{,}536{,}000 \\
& \qquad x_1, x_2 \in \{0,1\}
\end{aligned}
$$

The solution is: $x_1=1, x_2=0$ and $z_{lower}=37{,}536{,}000$.

Operation subproblem iteration 3:
Because of $r=0.0$ based on the above investment strategy, the trial schedule is feasible.

Operation subproblem iteration 3:
According to calculations, the primal solution of feasible subproblem is:

$w=31{,}536{,}000$

$f_{12}=-25,\ f_{13}=125,\ f_{23}=75,\ f_{24}=100,\ f_{34}=0\ \ \theta_1=0, \theta_2=2.5, \theta_3=-12.5, \theta_4=-17.5$

$g_1=100,\ g_2=200,\ g_3=100$

Because $z_{upper}=6{,}000{,}000+w=6{,}000{,}000+31{,}536{,}000=z_{lower}=37{,}536{,}000$, the optimal solution is obtained, which show that selecting the line 2-4 can satisfy the system curtailment requirement, though it has a higher investment cost than the candidate line 3-4.

MATHLAB FORMULATION:

```
function [X,FVAL,EXITFLAG,OUTPUT,LAMBDA] = LPModel

f=[0  0  0  0  0  0  0  0  0  0  0  0  1  1]; %% objective
A=[0  0  0  1  0  0 -5  0  5 0  0  0  0  0  %% line 2-4
   0  0  0 -1  0  0  5  0 -5 0  0  0  0  0  %% line 2-4
  ];
bT=[1000  1000];
b=transpose(bT);
Aeq=[-1 -1  0  0  0  0  0  0  0  1  0  0  0  0  %% bus 1
      1  0 -1 -1  0  0  0  0  0  0  1  0  0  0  %% bus 2
      0  1  1  0 -1  0  0  0  0  0  0  0  1  0  %% bus 3
      0  0  0  1  1  0  0  0  0  0  0  1  0  1  %% bus 4
      1  0  0  0  0 -10  10  0  0  0  0  0  0  0  %% line 1-2
      0  1  0  0  0 -10  0  10  0  0  0  0  0  0  %% line 1-3
      0  0  1  0  0  0 -5  5  0  0  0  0  0  0  %% line 2-3
      0  0  0  0  1  0  0 -5  5 0  0  0  0  0  %% line 3-4
    ];

beqT=[0  0  200  200  0  0  0  0];
beq=transpose(beqT);
LBT=[-150  -150  -100  0  -100  0  -Inf -Inf -Inf 50  100  50  0  0];
%% lower bound
```

```
LB=transpose(LBT);
UBT=[150  150  100  0  100  0  Inf  Inf  Inf  150  200  100  Inf
Inf]; %% upper bound
UB=transpose(UBT);

[X,FVAL,EXITFLAG,OUTPUT,LAMBDA]=LINPROG(f,A,b,Aeq,beq,LB,UB); %% LP
function


SOLUTION:

%% f, A, b, Aeq, beq, LB and UB matrix for each iteration

%%Feasibility check --- Iteration 1:

f=[0  0  0  0  0  0  0  0  0  0  0  0  1  1]; %% objective for one
hour
A=[0  0  0  1  0  0 -5  0  5 0  0  0  0  0  %% line 2-4
   0  0  0 -1  0  0  5  0 -5 0  0  0  0  0  %% line 2-4
  ];
bT=[1000  1000];
b=transpose(bT);
Aeq=[-1 -1  0  0  0  0  0  0  0  1  0  0  0  0  %% bus 1
      1  0 -1 -1  0  0  0  0  0  0  1  0  0  0  %% bus 2
      0  1  1  0 -1  0  0  0  0  0  0  0  1  0  %% bus 3
      0  0  0  1  1  0  0  0  0  0  0  1  0  1  %% bus 4
      1  0  0  0  0 -10  10  0  0  0  0  0  0  0  %% line 1-2
      0  1  0  0  0 -10  0  10  0  0  0  0  0  0  %% line 1-3
      0  0  1  0  0  0 -5  5  0  0  0  0  0  0  %% line 2-3
      0  0  0  0  1  0  0 -5  5 0  0  0  0  0  %% line 3-4
    ];
beqT=[0  0  200  200  0  0  0  0];
beq=transpose(beqT);
LBT=[-150  -150  -100  0  -100  0  -Inf -Inf -Inf 50  100  50  0  0];
%% lower bound
LB=transpose(LBT);
UBT=[150  150  100  0  100  0  Inf  Inf  Inf  150  200  100  Inf
Inf]; %% upper bound
UB=transpose(UBT);

%%Feasibility check --- Iteration 2

f=[0  0  0  0  0  0  0  0  0  0  0  0  1  1];  %% objective for one
hour
A=[0  0  0  0  1  0  0 -5  5 0  0  0  0  0  %% line 3-4
   0  0  0  0 -1  0  0  5 -5 0  0  0  0  0  %% line 3-4
  ];
bT=[1000  1000];
b=transpose(bT);
Aeq=[-1 -1  0  0  0  0  0  0  0  1  0  0  0  0  %% bus 1
      1  0 -1 -1  0  0  0  0  0  0  1  0  0  0  %% bus 2
      0  1  1  0 -1  0  0  0  0  0  0  0  1  0  %% bus 3
```

```
      0  0  0  1  1  0  0  0  0  0  0  1  0  1  %% bus 4
      1  0  0  0  0 -10  10  0  0  0  0  0  0  0  %% line 1-2
      0  1  0  0  0 -10  0  10  0  0  0  0  0  0  %% line 1-3
      0  0  1  0  0  0 -5  5  0  0  0  0  0  0  %% line 2-3
      0  0  0  1  0  0 -5  0  5 0  0  0  0  0  %% line 2-4
    ];
beqT=[0  0  200  200  0  0  0  0];
beq=transpose(beqT);
LBT=[-150  -150  -100  -100  0  0  -Inf -Inf -Inf 50  100  50  0  0];
%% lower bound
LB=transpose(LBT);
UBT=[150  150  100  100  0  0  Inf  Inf  Inf  150  200  100  Inf
Inf];  %% upper bound
UB=transpose(UBT);

%%Optimal operation --- Iteration 2

f=[0  0  0  0  0  0  0  0  0  10  8  10];  %% objective for one hour
A=[0  0  0  0  1  0  0 -5  5 0  0  0 %% line 3-4
   0  0  0  0 -1  0  0  5 -5 0  0  0 %% line 3-4
  ];
bT=[1000  1000];
b=transpose(bT);
Aeq=[-1 -1  0  0  0  0  0  0  0  1  0  0  %% bus 1
      1  0 -1 -1  0  0  0  0  0  0  1  0  %% bus 2
      0  1  1  0 -1  0  0  0  0  0  0  0  %% bus 3
      0  0  0  1  1  0  0  0  0  0  0  1  %% bus 4
      1  0  0  0  0 -10  10  0  0  0  0  0  %% line 1-2
      0  1  0  0  0 -10  0  10  0  0  0  0  %% line 1-3
      0  0  1  0  0  0 -5  5  0  0  0  0  %% line 2-3
      0  0  0  1  0  0 -5  0  5 0  0  0  %% line 2-4
    ];
beqT=[0  0  200  200  0  0  0  0];
beq=transpose(beqT);
LBT=[-150  -150  -100  -100  0  0  -Inf -Inf -Inf 50  100  50]; %%
lower bound
LB=transpose(LBT);
UBT=[150  150  100  100  0  0  Inf  Inf  Inf  150  200  100]; %% upper
bound
UB=transpose(UBT);

%%Feasibility check --- Iteration 3

f=[0  0  0  0  0  0  0  0  0  0  0  0  1  1];  %% objective for one
hour
A=[0  0  0  0  1  0  0 -5  5 0  0  0  0  0  %% line 3-4
   0  0  0  0 -1  0  0  5 -5 0  0  0  0  0  %% line 3-4
  ];
bT=[1000  1000];
b=transpose(bT);
Aeq=[-1 -1  0  0  0  0  0  0  0  1  0  0  0  0  %% bus 1
      1  0 -1 -1  0  0  0  0  0  0  1  0  0  0  %% bus 2
```

```
      0  1  1  0 -1  0  0  0  0  0  0  0  1  0  %% bus 3
      0  0  0  1  1  0  0  0  0  0  0  1  0  1  %% bus 4
      1  0  0  0  0 -10  10  0  0  0  0  0  0  0  %% line 1-2
      0  1  0  0  0 -10  0  10  0  0  0  0  0  0  %% line 1-3
      0  0  1  0  0  0 -5  5  0  0  0  0  0  0  %% line 2-3
      0  0  0  1  0  0 -5  0  5 0  0  0  0  0  %% line 2-4
    ];
beqT=[0  0  200  200  0  0  0  0];
beq=transpose(beqT);
LBT=[-150  -150  -100  -100  0  0  -Inf -Inf -Inf 50  100  50  0  0];
%% lower bound
LB=transpose(LBT);
UBT=[150  150  100  100  0  0  Inf  Inf  Inf  150  200  100  Inf
Inf];  %% upper bound
UB=transpose(UBT);

%%Optimal operation --- Iteration 3

f=[0  0  0  0  0  0  0  0  0  10  8  10];  %% objective for one hour
A=[0  0  0  0  1  0  0 -5  5 0  0  0 %% line 3-4
   0  0  0  0 -1  0  0  5 -5 0  0  0 %% line 3-4
  ];
bT=[1000  1000];
b=transpose(bT);
Aeq=[-1 -1  0  0  0  0  0  0  0  1  0  0  %% bus 1
      1  0 -1 -1  0  0  0  0  0  0  1  0  %% bus 2
      0  1  1  0 -1  0  0  0  0  0  0  0  %% bus 3
      0  0  0  1  1  0  0  0  0  0  0  1  %% bus 4
      1  0  0  0  0 -10  10  0  0  0  0  0  %% line 1-2
      0  1  0  0  0 -10  0  10  0  0  0  0  %% line 1-3
      0  0  1  0  0  0 -5  5  0  0  0  0  %% line 2-3
      0  0  0  1  0  0 -5  0  5 0  0  0  %% line 2-4
    ];
beqT=[0  0  200  200  0  0  0  0];
beq=transpose(beqT);
LBT=[-150  -150  -100  -100  0  0  -Inf -Inf -Inf 50  100  50]; %%
lower bound
LB=transpose(LBT);
UBT=[150  150  100  100  0  0  Inf  Inf  Inf  150  200  100]; %% upper
bound
UB=transpose(UBT);
```

8.1 IMPACT OF CONTINGENCIES ON TRANSMISSION PLANNING:

If we consider the N-1 contingencies, the solution framework is shown as follows:

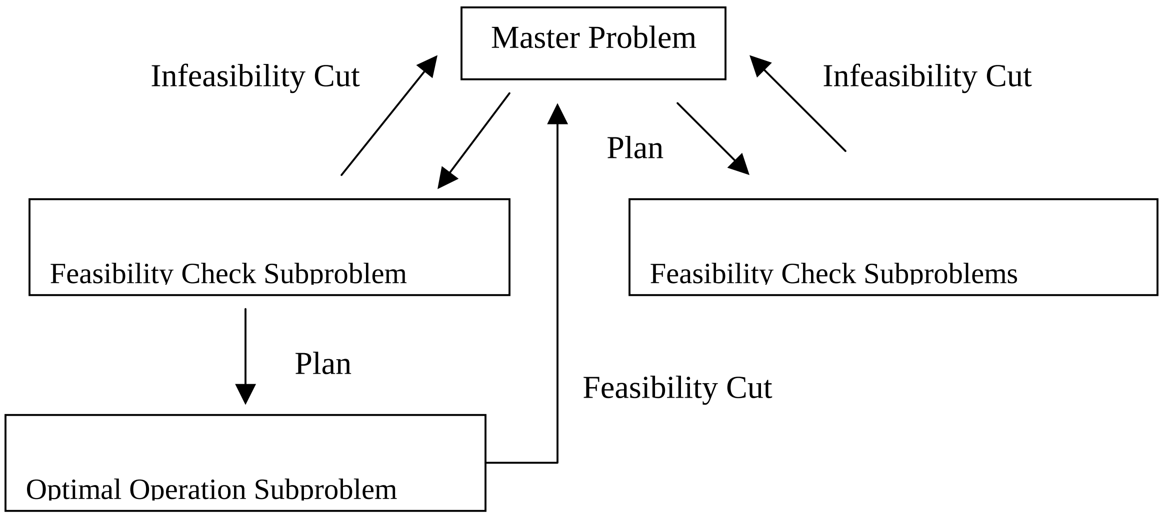

Figure 1 Solution Framework

Example 2

A 4-bus system is shown in Figure 2. Bus 1 is the slack bus. Existing and candidate line, generator, load data are given in Tables 4 through 6. We assume the studied planning period only has one-year interval. Loads are assumed constant during the period. At least one among three candidate lines should be built to transfer the electricity to the new load bus in the planning year 1. The maximum energy not served requirement (ε) is 0 p.u. under the steady state and any single-line outages (N-1 checking principle) in the planning year 1.

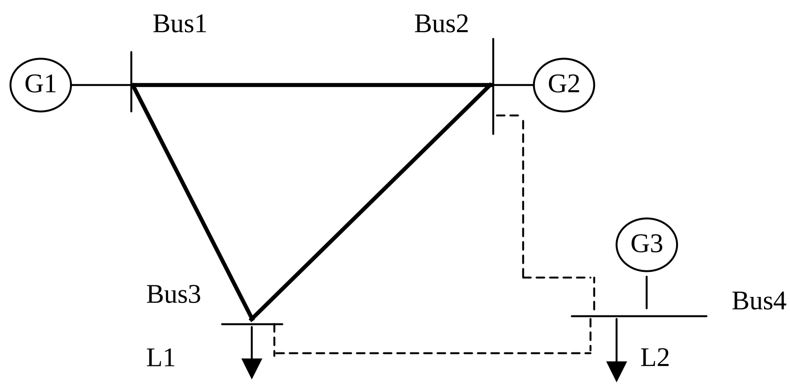

Figure 2 Three-Bus System Example

Table 4 Existing and candidate line Data for the 4-bus System

Line	From bus	To bus	Reactance (pu)	Capacity (MW)	Investment cost ($)
1	2	4	0.2	150	6,000,000
2	3	4	0.2	150	5,000,000
3	1	2	0.1	200	-
4	2	3	0.2	200	-
5	1	3	0.1	200	-

Table 5 Generator Data for the 4-bus System

Unit	Min Capacity (MW)	Max Capacity (MW)	Cost ($/MWh)
1	100	200	8
2	100	200	8
3	50	100	10

Table 6 Load Data (MW)

Planning Year	L1	L2
1	200	200

The original objective is $Min\ 6{,}000{,}000*(x_1-0)+5{,}000{,}000*(x_2-0)+8760*(10g_1+8g_2+10g_3)$

where x_1 represents the state of candidate line 2-4 and x_2 represents the state of candidate line 3-4.

First, we solve initial transmission planning master problem.

Transmission planning master problem iteration 1:

$$
\begin{aligned}
&Min\ z_{lower} \\
&S.t. \quad z_{lower} \ge 6{,}000{,}000*(x_1-0)+5{,}000{,}000*(x_2-0) \\
&\qquad x_1+x_2 \ge 1 \\
&\qquad x_1, x_2 \in \{0,1\}
\end{aligned}
$$

The solution is: $x_1=0, x_2=1$ and $z_{lower}=5{,}000{,}000$.

Operation subproblem under the steady state iteration 1:

We check the feasibility of operation subproblem given the first trial of transmission planning schedule. The feasibility check is as follows:

$$
\begin{aligned}
& Min\ 8760*(r_1 + r_2) \\
& S.t. - f_{12} - f_{13} + g_1 = 0 \\
& \quad - f_{23} + f_{12} - f_{24} + g_2 = 0 \\
& \quad f_{13} + f_{23} - f_{34} + r_1 = 200 \\
& \quad f_{24} + f_{34} + g_3 + r_2 = 200 \\
& \quad f_{12} - {(\theta_1 - \theta_2)}/{0.1} = 0 \\
& \quad f_{13} - {(\theta_1 - \theta_3)}/{0.1} = 0 \\
& \quad f_{23} - {(\theta_2 - \theta_3)}/{0.2} = 0 \\
& \quad f_{24} - {(\theta_2 - \theta_4)}/{0.2} \le 1000*(1 - \hat{x}_1) \qquad \overline{\pi}_1 \\
& \quad -(f_{24} - {(\theta_2 - \theta_4)}/{0.2}) \le 1000*(1 - \hat{x}_1) \qquad \underline{\pi}_1 \\
& \quad f_{34} - {(\theta_3 - \theta_4)}/{0.2} \le 1000*(1 - \hat{x}_2) \qquad \overline{\pi}_2 \\
& \quad -(f_{34} - {(\theta_3 - \theta_4)}/{0.2}) \le 1000*(1 - \hat{x}_2) \qquad \underline{\pi}_2 \\
& \quad -200 \le f_{12} \le 200 \\
& \quad -200 \le f_{13} \le 200 \\
& \quad -200 \le f_{23} \le 200 \\
& \quad f_{24} \le 150*\hat{x}_1 \qquad \overline{\lambda}_1 \\
& \quad - f_{24} \le 150*\hat{x}_1 \qquad \underline{\lambda}_1 \\
& \quad f_{34} \le 150*\hat{x}_2 \qquad \overline{\lambda}_2 \\
& \quad - f_{34} \le 150*\hat{x}_2 \qquad \underline{\lambda}_2 \\
& \quad 100 \le g_1 \le 200 \\
& \quad 100 \le g_2 \le 200 \\
& \quad 50 \le g_3 \le 100 \\
& \quad \theta_1 = 0
\end{aligned}
$$

Because $r = 0.0$ based on the above investment strategy, the trial schedule is feasible.

Operation subproblem under the line 1-2 outage iteration 1:

The feasibility check is as follows:

$$Min\ 8760*(r_1+r_2)$$

$$S.t.\quad f_{13}+g_1=0$$

$$-f_{23}-f_{24}+g_2=0$$

$$f_{13}+f_{23}-f_{34}+r_1=200$$

$$f_{24}+f_{34}+g_3+r_2=200$$

$$f_{13}-(\theta_1-\theta_3)/0.1=0$$

$$f_{23}-(\theta_2-\theta_3)/0.2=0$$

$$f_{24}-(\theta_2-\theta_4)/0.2\le 1000*(1-\hat{x}_1)\qquad \overline{\pi}_1$$

$$-(f_{24}-(\theta_2-\theta_4)/0.2)\le 1000*(1-\hat{x}_1)\qquad \underline{\pi}_1$$

$$f_{34}-(\theta_3-\theta_4)/0.2\le 1000*(1-\hat{x}_2)\qquad \overline{\pi}_2$$

$$-(f_{34}-(\theta_3-\theta_4)/0.2)\le 1000*(1-\hat{x}_2)\qquad \underline{\pi}_2$$

$$-200\le f_{13}\le 200$$

$$-200\le f_{23}\le 200$$

$$f_{24}\le 150*\hat{x}_1\qquad \overline{\lambda}_1$$

$$-f_{24}\le 150*\hat{x}_1\qquad \underline{\lambda}_1$$

$$f_{34}\le 150*\hat{x}_2\qquad \overline{\lambda}_2$$

$$-f_{34}\le 150*\hat{x}_2\qquad \underline{\lambda}_2$$

$$100\le g_1\le 200$$

$$100\le g_2\le 200$$

$$50\le g_3\le 100$$

$$\theta_1=0$$

Because $r=0.0$, the trial schedule is feasible.

Operation subproblem under the line 1-3 outage iteration 1:

The feasibility check is as follows:

$$Min\ 8760*(r_1+r_2)$$

$$S.t.-f_{12}+g_1=0$$

$$-f_{23}+f_{12}-f_{24}+g_2=0$$

$$f_{23}-f_{34}+r_1=200$$

$$f_{24}+f_{34}+g_3+r_2=200$$

$$f_{12}-(\theta_1-\theta_2)/0.1=0$$

$$f_{23}-(\theta_2-\theta_3)/0.2=0$$

$$f_{24} - (\theta_2 - \theta_4)/0.2 \le 1000*(1-\hat{x}_1) \qquad \overline{\pi}_1$$
$$-(f_{24} - (\theta_2 - \theta_4)/0.2) \le 1000*(1-\hat{x}_1) \qquad \underline{\pi}_1$$
$$f_{34} - (\theta_3 - \theta_4)/0.2 \le 1000*(1-\hat{x}_2) \qquad \overline{\pi}_2$$
$$-(f_{34} - (\theta_3 - \theta_4)/0.2) \le 1000*(1-\hat{x}_2) \qquad \underline{\pi}_2$$
$$-200 \le f_{12} \le 200$$
$$-200 \le f_{23} \le 200$$
$$f_{24} \le 150*\hat{x}_1 \qquad \overline{\lambda}_1$$
$$-f_{24} \le 150*\hat{x}_1 \qquad \underline{\lambda}_1$$
$$f_{34} \le 150*\hat{x}_2 \qquad \overline{\lambda}_2$$
$$-f_{34} \le 150*\hat{x}_2 \qquad \underline{\lambda}_2$$
$$100 \le g_1 \le 200$$
$$100 \le g_2 \le 200$$
$$50 \le g_3 \le 100$$
$$\theta_1 = 0$$

Because $r = 0.0$, the trial schedule is feasible.

<u>Operation subproblem under the line 2-3 outage iteration 1:</u>

The feasibility check is as follows:

$$Min\ 8760*(r_1 + r_2)$$
$$S.t. -f_{12} - f_{13} + g_1 = 0$$
$$f_{12} - f_{24} + g_2 = 0$$
$$f_{13} - f_{34} + r_1 = 200$$
$$f_{24} + f_{34} + g_3 + r_2 = 200$$
$$f_{12} - (\theta_1 - \theta_2)/0.1 = 0$$
$$f_{13} - (\theta_1 - \theta_3)/0.1 = 0$$
$$f_{24} - (\theta_2 - \theta_4)/0.2 \le 1000*(1-\hat{x}_1) \qquad \overline{\pi}_1$$
$$-(f_{24} - (\theta_2 - \theta_4)/0.2) \le 1000*(1-\hat{x}_1) \qquad \underline{\pi}_1$$
$$f_{34} - (\theta_3 - \theta_4)/0.2 \le 1000*(1-\hat{x}_2) \qquad \overline{\pi}_2$$
$$-(f_{34} - (\theta_3 - \theta_4)/0.2) \le 1000*(1-\hat{x}_2) \qquad \underline{\pi}_2$$
$$-200 \le f_{12} \le 200$$
$$-200 \le f_{13} \le 200$$
$$f_{24} \le 150*\hat{x}_1 \qquad \overline{\lambda}_1$$
$$-f_{24} \le 150*\hat{x}_1 \qquad \underline{\lambda}_1$$
$$f_{34} \le 150*\hat{x}_2 \qquad \overline{\lambda}_2$$
$$-f_{34} \le 150*\hat{x}_2 \qquad \underline{\lambda}_2$$

$$100 \le g_1 \le 200$$
$$100 \le g_2 \le 200$$
$$50 \le g_3 \le 100$$
$$\theta_1 = 0$$

Because $r = 0.0$, the trial schedule is feasible.

<u>Operation subproblem under the line 3-4 outage iteration 1:</u>

The feasibility check is as follows:

$$Min\ 8760*(r_1 + r_2)$$
$$S.t. - f_{12} - f_{13} + g_1 = 0$$
$$- f_{23} + f_{12} - f_{24} + g_2 = 0$$
$$f_{13} + f_{23} + r_1 = 200$$
$$f_{24} + g_3 + r_2 = 200$$
$$f_{12} - {(\theta_1 - \theta_2)}/{0.1} = 0$$
$$f_{13} - {(\theta_1 - \theta_3)}/{0.1} = 0$$
$$f_{23} - {(\theta_2 - \theta_3)}/{0.2} = 0$$
$$f_{24} - {(\theta_2 - \theta_4)}/{0.2} \le 1000*(1 - \hat{x}_1) \qquad \overline{\pi}_1$$
$$-(f_{24} - {(\theta_2 - \theta_4)}/{0.2}) \le 1000*(1 - \hat{x}_1) \qquad \underline{\pi}_1$$
$$-200 \le f_{12} \le 200$$
$$-200 \le f_{13} \le 200$$
$$-200 \le f_{23} \le 200$$
$$f_{24} \le 150*\hat{x}_1 \qquad \overline{\lambda}_1$$
$$- f_{24} \le 150*\hat{x}_1 \qquad \underline{\lambda}_1$$
$$100 \le g_1 \le 200$$
$$100 \le g_2 \le 200$$
$$50 \le g_3 \le 100$$
$$\theta_1 = 0$$

The primal solutions of feasibility check are

$f_{12} = -25.0,\ f_{13} = 125.0,\ f_{23} = 75,\ f_{24} = 0,\ f_{34} = 0\ \ \theta_1 = 0, \theta_2 = 2.5, \theta_3 = -12.5, \theta_4 = 2.2010$

$g_1 = 100, g_2 = 100, g_3 = 100$ and $r = r_1 + r_2 = 0 + 100 = 100$, which means the trial schedule is infeasible for the line 3-4 outage. The dual multipliers of the operation subproblem are:

$\overline{\lambda}_1 = 8760*(-1) \quad \underline{\lambda}_1 = 0$

$\overline{\pi}_1 = 0 \quad \underline{\pi}_1 = 0$

So, the infeasibility cut is as

$$8760*r + (\overline{\lambda}_1 + \underline{\lambda}_1)*150*(x_1 - \hat{x}_1) - (\overline{\pi}_1 + \underline{\pi}_1)*1000*(x_1 - \hat{x}_1) \le 0$$
$$\Rightarrow$$
$$876{,}000 - 1{,}314{,}000*(x_1 - 0) \le 0$$

Transmission planning master problem iteration 2:

$$\begin{aligned}
&Min\ z_{lower} \\
&S.t.\quad z_{lower} \geq 6{,}000{,}000*(x_1-0)+5{,}000{,}000*(x_2-0) \\
&\qquad x_1+x_2 \geq 1 \\
&\qquad 876{,}000-1{,}314{,}000*(x_1-0) \leq 0 \\
&\qquad x_1, x_2 \in \{0,1\}
\end{aligned}$$

The solution is: $x_1=1, x_2=0$ and $z_{lower}=6{,}000{,}000$.

Operation subproblem under the steady state iteration 2:
The feasibility check is as follows:

$$\begin{aligned}
&Min\ 8760*(r_1+r_2) \\
&S.t. - f_{12} - f_{13} + g_1 = 0 \\
&\quad - f_{23} + f_{12} - f_{24} + g_2 = 0 \\
&\quad f_{13} + f_{23} - f_{34} + r_1 = 200 \\
&\quad f_{24} + f_{34} + g_3 + r_2 = 200 \\
&\quad f_{12} - (\theta_1-\theta_2)/0.1 = 0 \\
&\quad f_{13} - (\theta_1-\theta_3)/0.1 = 0 \\
&\quad f_{23} - (\theta_2-\theta_3)/0.2 = 0 \\
&\quad f_{24} - (\theta_2-\theta_4)/0.2 \leq 1000*(1-\hat{x}_1) \qquad \overline{\pi}_1 \\
&\quad -(f_{24} - (\theta_2-\theta_4)/0.2) \leq 1000*(1-\hat{x}_1) \qquad \underline{\pi}_1 \\
&\quad f_{34} - (\theta_3-\theta_4)/0.2 \leq 1000*(1-\hat{x}_2) \qquad \overline{\pi}_2 \\
&\quad -(f_{34} - (\theta_3-\theta_4)/0.2) \leq 1000*(1-\hat{x}_2) \qquad \underline{\pi}_2 \\
&\quad -200 \leq f_{12} \leq 200 \\
&\quad -200 \leq f_{13} \leq 200 \\
&\quad -200 \leq f_{23} \leq 200 \\
&\quad f_{24} \leq 150*\hat{x}_1 \qquad \overline{\lambda}_1 \\
&\quad - f_{24} \leq 150*\hat{x}_1 \qquad \underline{\lambda}_1 \\
&\quad f_{34} \leq 150*\hat{x}_2 \qquad \overline{\lambda}_2 \\
&\quad - f_{34} \leq 150*\hat{x}_2 \qquad \underline{\lambda}_2 \\
&\quad 100 \leq g_1 \leq 200 \\
&\quad 100 \leq g_2 \leq 200 \\
&\quad 50 \leq g_3 \leq 100 \\
&\quad \theta_1 = 0
\end{aligned}$$

Because $r=0.0$ based on the above investment strategy, the trial schedule is feasible.

Operation subproblem under the line 1-2 outage iteration 2:

The feasibility check is as follows:

$$Min\ 8760*(r_1+r_2)$$

$$S.t.\quad f_{13}+g_1=0$$

$$-f_{23}-f_{24}+g_2=0$$

$$f_{13}+f_{23}-f_{34}+r_1=200$$

$$f_{24}+f_{34}+g_3+r_2=200$$

$$f_{13}-(\theta_1-\theta_3)/0.1 \quad =0$$

$$f_{23}-(\theta_2-\theta_3)/0.2 \quad =0$$

$$f_{24}-(\theta_2-\theta_4)/0.2 \quad \le 1000*(1-\hat{x}_1) \qquad \overline{\pi}_1$$

$$-(f_{24}-(\theta_2-\theta_4)/0.2) \le 1000*(1-\hat{x}_1) \qquad \underline{\pi}_1$$

$$f_{34}-(\theta_3-\theta_4)/0.2 \quad \le 1000*(1-\hat{x}_2) \qquad \overline{\pi}_2$$

$$-(f_{34}-(\theta_3-\theta_4)/0.2) \le 1000*(1-\hat{x}_2) \qquad \underline{\pi}_2$$

$$-200\le f_{13}\le 200$$

$$-200\le f_{23}\le 200$$

$$f_{24}\le 150*\hat{x}_1 \qquad \overline{\lambda}_1$$

$$-f_{24}\le 150*\hat{x}_1 \qquad \underline{\lambda}_1$$

$$f_{34}\le 150*\hat{x}_2 \qquad \overline{\lambda}_2$$

$$-f_{34}\le 150*\hat{x}_2 \qquad \underline{\lambda}_2$$

$$100\le g_1\le 200$$

$$100\le g_2\le 200$$

$$50\le g_3\le 100$$

$$\theta_1=0$$

Because $r=0.0$, the trial schedule is feasible.

Operation subproblem under the line 1-3 outage iteration 2:

The feasibility check is as follows:

$$Min\ 8760*(r_1+r_2)$$

$$S.t. -f_{12}+g_1=0$$

$$-f_{23}+f_{12}-f_{24}+g_2=0$$

$$f_{23}-f_{34}+r_1=200$$

$$f_{24}+f_{34}+g_3+r_2=200$$

$$f_{12}-(\theta_1-\theta_2)/0.1=0$$

$$f_{23}-(\theta_2-\theta_3)/0.2=0$$

$$f_{24}-(\theta_2-\theta_4)/0.2\le 1000*(1-\hat{x}_1)\qquad \overline{\pi}_1$$

$$-(f_{24}-(\theta_2-\theta_4)/0.2)\le 1000*(1-\hat{x}_1)\qquad \underline{\pi}_1$$

$$f_{34}-(\theta_3-\theta_4)/0.2\le 1000*(1-\hat{x}_2)\qquad \overline{\pi}_2$$

$$-(f_{34}-(\theta_3-\theta_4)/0.2)\le 1000*(1-\hat{x}_2)\qquad \underline{\pi}_2$$

$$-200\le f_{12}\le 200$$

$$-200\le f_{23}\le 200$$

$$f_{24}\le 150*\hat{x}_1\qquad \overline{\lambda}_1$$

$$-f_{24}\le 150*\hat{x}_1\qquad \underline{\lambda}_1$$

$$f_{34}\le 150*\hat{x}_2\qquad \overline{\lambda}_2$$

$$-f_{34}\le 150*\hat{x}_2\qquad \underline{\lambda}_2$$

$$100\le g_1\le 200$$

$$100\le g_2\le 200$$

$$50\le g_3\le 100$$

$$\theta_1=0$$

Because $r=0.0$, the trial schedule is feasible.

Operation subproblem under the line 2-3 outage iteration 2:

The feasibility check is as follows:

$$Min\ 8760*(r_1+r_2)$$
$$S.t. -f_{12}-f_{13}+g_1=0$$
$$f_{12}-f_{24}+g_2=0$$
$$f_{13}-f_{34}+r_1=200$$
$$f_{24}+f_{34}+g_3+r_2=200$$
$$f_{12}-{(\theta_1-\theta_2)}/{0.1}=0$$
$$f_{13}-{(\theta_1-\theta_3)}/{0.1}=0$$
$$f_{24}-{(\theta_2-\theta_4)}/{0.2}\le 1000*(1-\hat{x}_1) \qquad \overline{\pi}_1$$
$$-(f_{24}-{(\theta_2-\theta_4)}/{0.2})\le 1000*(1-\hat{x}_1) \qquad \underline{\pi}_1$$
$$f_{34}-{(\theta_3-\theta_4)}/{0.2}\le 1000*(1-\hat{x}_2) \qquad \overline{\pi}_2$$
$$-(f_{34}-{(\theta_3-\theta_4)}/{0.2})\le 1000*(1-\hat{x}_2) \qquad \underline{\pi}_2$$
$$-200\le f_{12}\le 200$$
$$-200\le f_{13}\le 200$$
$$f_{24}\le 150*\hat{x}_1 \qquad \overline{\lambda}_1$$
$$-f_{24}\le 150*\hat{x}_1 \qquad \underline{\lambda}_1$$
$$f_{34}\le 150*\hat{x}_2 \qquad \overline{\lambda}_2$$
$$-f_{34}\le 150*\hat{x}_2 \qquad \underline{\lambda}_2$$
$$100\le g_1\le 200$$
$$100\le g_2\le 200$$
$$50\le g_3\le 100$$
$$\theta_1=0$$

Because $r=0.0$, the trial schedule is feasible.

Operation subproblem under the line 2-4 outage iteration 2:
The feasibility check is as follows:

$$
\begin{aligned}
& Min\ 8760*(r_1+r_2) \\
& S.t. -f_{12}-f_{13}+g_1=0 \\
& \quad -f_{23}+f_{12}+g_2=0 \\
& \quad f_{13}+f_{23}-f_{34}+r_1=200 \\
& \quad f_{34}+g_3+r_2=200 \\
& \quad f_{12}-{(\theta_1-\theta_2)}/{0.1}=0 \\
& \quad f_{13}-{(\theta_1-\theta_3)}/{0.1}=0 \\
& \quad f_{23}-{(\theta_2-\theta_3)}/{0.2}=0 \\
& \quad f_{34}-{(\theta_3-\theta_4)}/{0.2}\le 1000*(1-\hat{x}_2) \qquad \overline{\pi}_2 \\
& \quad -(f_{34}-{(\theta_3-\theta_4)}/{0.2})\le 1000*(1-\hat{x}_2) \qquad \underline{\pi}_2 \\
& \quad -200\le f_{12}\le 200 \\
& \quad -200\le f_{13}\le 200 \\
& \quad -200\le f_{23}\le 200 \\
& \quad f_{34}\le 150*\hat{x}_2 \qquad \overline{\lambda}_2 \\
& \quad -f_{34}\le 150*\hat{x}_2 \qquad \underline{\lambda}_2 \\
& \quad 100\le g_1\le 200 \\
& \quad 100\le g_2\le 200 \\
& \quad 50\le g_3\le 100 \\
& \quad \theta_1=0
\end{aligned}
$$

The primal solutions of feasibility check are
$f_{12}=-25.0,\ f_{13}=125.0,\ f_{23}=75,\ f_{24}=0,\ f_{34}=0\ \ \theta_1=0, \theta_2=2.5, \theta_3=-12.5, \theta_4=-12.3346$
$g_1=100, g_2=100, g_3=100$ and $r=r_1+r_2=0+100=100$, which means the trial schedule is infeasible for the line 2-4 outage. The dual multipliers of the operation subproblem are:

$\overline{\lambda}_2=8760*(-1) \quad \underline{\lambda}_2=0$

$\overline{\pi}_2=0 \quad \underline{\pi}_2=0$

Therefore, the infeasibility cut is as

$$8760*r+(\overline{\lambda}_2+\underline{\lambda}_2)*150*(x_2-\hat{x}_2)-(\overline{\pi}_2+\underline{\pi}_2)*1000*(x_2-\hat{x}_2)\le 0$$

$$\Rightarrow$$

$$876{,}000-1{,}314{,}000*(x_2-0)\le 0$$

Transmission planning master problem iteration 3:

$$
\begin{aligned}
&Min\ z_{lower} \\
&S.t. \quad z_{lower} \ge 6{,}000{,}000 * (x_1 - 0) + 5{,}000{,}000 * (x_2 - 0) \\
&\quad x_1 + x_2 \ge 1 \\
&\quad 876{,}000 - 1{,}314{,}000 * (x_1 - 0) \le 0 \\
&\quad 876{,}000 - 1{,}314{,}000 * (x_2 - 0) \le 0 \\
&\quad x_1, x_2 \in \{0, 1\}
\end{aligned}
$$

The solution is: $x_1 = 1, x_2 = 1$ and $z_{lower} = 11{,}000{,}000$.

Operation subproblems under the steady state and any single-line outage iteration 3:

According to feasibility checks for the steady and any single-line outage, all $r = 0.0$. Thus, the trial schedule is feasible.

Operation subproblem under the steady state iteration 3:

The feasible subproblem is as follows

$$
\begin{aligned}
&Min\ 8760 * (10 * g_1 + 8 * g_2 + 10 * g_3) \\
&S.t. - f_{12} - f_{13} + g_1 = 0 \\
&\quad - f_{23} + f_{12} - f_{24} + g_2 = 0 \\
&\quad f_{13} + f_{23} - f_{34} = 200 \\
&\quad f_{24} + f_{34} + g_3 = 200 \\
&\quad f_{12} - (\theta_1 - \theta_2)/0.1 = 0 \\
&\quad f_{13} - (\theta_1 - \theta_3)/0.1 = 0 \\
&\quad f_{23} - (\theta_2 - \theta_3)/0.2 = 0 \\
&\quad f_{24} - (\theta_2 - \theta_4)/0.2 \le 1000 * (1 - \hat{x}_1) \qquad \overline{\beta}_1 \\
&\quad -(f_{24} - (\theta_2 - \theta_4)/0.2) \le 1000 * (1 - \hat{x}_1) \qquad \underline{\beta}_1 \\
&\quad f_{34} - (\theta_3 - \theta_4)/0.2 \le 1000 * (1 - \hat{x}_2) \qquad \overline{\beta}_2 \\
&\quad -(f_{34} - (\theta_3 - \theta_4)/0.2) \le 1000 * (1 - \hat{x}_2) \qquad \underline{\beta}_2 \\
&\quad -200 \le f_{12} \le 200 \\
&\quad -200 \le f_{13} \le 200 \\
&\quad -200 \le f_{23} \le 200 \\
&\quad f_{24} \le 150 * \hat{x}_1 \qquad \overline{\alpha}_1 \\
&\quad - f_{24} \le 150 * \hat{x}_1 \qquad \underline{\alpha}_1 \\
&\quad f_{34} \le 150 * \hat{x}_2 \qquad \overline{\alpha}_2 \\
&\quad - f_{34} \le 150 * \hat{x}_2 \qquad \underline{\alpha}_2 \\
&\quad 100 \le g_1 \le 200 \\
&\quad 100 \le g_2 \le 200 \\
&\quad 50 \le g_3 \le 100 \\
&\quad \theta_1 = 0
\end{aligned}
$$

The primal solutions of feasible subproblem are:

$w = 31{,}536{,}000$
$f_{12} = -14.7334,\ f_{13} = 145.266,\ f_{23} = 80.0,\ f_{24} = 105.2666,\ f_{34} = 25.2666$
$\theta_1 = 0, \theta_2 = 1.4733, \theta_3 = -14.5267, \theta_4 = -19.58$
$g_1 = 130.5331, g_2 = 200, g_3 = 69.4669$

The dual multipliers of the operation subproblem are:
$\overline{\alpha}_1 = 0 \quad \underline{\alpha}_1 = 0 \quad \overline{\alpha}_2 = 0 \quad \underline{\alpha}_2 = 0$
$\overline{\beta}_1 = 0 \quad \underline{\beta}_1 = 0 \quad \overline{\beta}_2 = 0 \quad \underline{\beta}_2 = 0$

Because $z_{upper} = 11{,}000{,}000 + w = 11{,}000{,}000 + 31{,}536{,}000 = 42{,}536{,}000 > z_{lower} = 11{,}000{,}000$, the feasible cut for the second iteration is:

$$\begin{aligned}z_{lower} &\geq 6{,}000{,}000*(x_1 - 0) + 5{,}000{,}000*(x_2 - 0) + 31{,}536{,}000 \\ &+ (\overline{\alpha}_1 + \underline{\alpha}_1)*150*(x_1 - \hat{x}_1) + (\overline{\alpha}_2 + \underline{\alpha}_2)*150*(x_2 - \hat{x}_2) \\ &- (\overline{\beta}_1 + \underline{\beta}_1)*1000*(x_1 - \hat{x}_1) - (\overline{\beta}_2 + \underline{\beta}_2)*1000*(x_2 - \hat{x}_2)\end{aligned}$$
$\Rightarrow$
$$z_{lower} \geq 6{,}000{,}000*(x_1 - 0) + 5{,}000{,}000*(x_2 - 0) + 31{,}536{,}000$$

<u>Transmission planning master problem iteration 4:</u>

$$\begin{aligned}&Min\ z_{lower} \\ &S.t. \quad z_{lower} \geq 6{,}000{,}000*(x_1 - 0) + 5{,}000{,}000*(x_2 - 0) \\ &\qquad x_1 + x_2 \geq 1 \\ &\qquad 876{,}000 - 1{,}314{,}000*(x_1 - 0) \leq 0 \\ &\qquad 876{,}000 - 1{,}314{,}000*(x_2 - 0) \leq 0 \\ &\qquad z_{lower} \geq 6{,}000{,}000*(x_1 - 0) + 5{,}000{,}000*(x_2 - 0) + 31{,}536{,}000 \\ &\qquad x_1, x_2 \in \{0, 1\} \\ &\qquad x_1, x_2 \in \{0, 1\}\end{aligned}$$

The solution is: $x_1 = 1, x_2 = 1$ and $z_{lower} = 42{,}536{,}000$.

<u>Operation subproblems under the steady state and any single-line outage iteration 4:</u>
Because all $r = 0.0$ based on the above investment strategy, the trial schedule is feasible.

<u>Operation subproblem under the steady state iteration 4:</u>
According to calculations, the primal solution of feasible subproblem is:
$w = 31{,}536{,}000$
$f_{12} = -14.7334,\ f_{13} = 145.266,\ f_{23} = 80.0,\ f_{24} = 105.2666,\ f_{34} = 25.2666$
$\theta_1 = 0, \theta_2 = 1.4733, \theta_3 = -14.5267, \theta_4 = -19.58$
$g_1 = 130.5331, g_2 = 200, g_3 = 69.4669$

Because $z_{upper} = 11{,}000{,}000 + w = 11{,}000{,}000 + 31{,}536{,}000 = z_{lower} = 42{,}536{,}000$, the optimal solution is obtained which show that lines 2-4 and 3-4 can satisfy the system curtailment requirement under the steady state and any single-line outages.

8. Further Reading

- M. Shahidehpour, H. Yamin and Z.Y. Li, Market Operations in Electric Power Systems, John Wiley &Sons, Inc., New York, 2002.
- M. Shahidehpour and M. Marwali, Maintenance Scheduling in Restructured Power Systems, Kluwer Academic Publishers, Norwell, Massachusetts, 2000.
- A. Monticelli, M. Pereira and S. Granville, “Security-Constrained Optimal Power Flow with Post-Contingency Corrective Rescheduling,” IEEE Trans. on Power Systems, Vol. 2, No. 2, pp.175-182, Feb. 1987.
- A.M. Geoffrion, "Generalized Benders Decomposition,” Journal of Optimization Theory and Applications, Vol. 10, no.4, pp. 237-261, 1972.
- J. F. Benders, “Partitioning Methods for Solving Mixed Variables Programming Problems,” Numerische Mathematik, Vol. 4, pp. 238–252, 1962.