

Stochastic Optimal Power Flow Analyses for Vieques and Culebra Island Microgrids with Variable PV Generation

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Abstract—This article discusses the challenges faced by LUMA's electric power grid in Puerto Rico's islands of Vieques and Culebra when dealing with the variability of photovoltaic (PV) power penetration. The focus of this work is to introduce a stochastic optimal power flow solution for managing the unbalanced three-phase distribution system in the two islands. The study provides a comprehensive grid reliability analyses for considering aggregated and distributed PV generation in Vieques and Culebra microgrids. By examining the implications of PV power penetration on the local distribution grid in Puerto Rico, this article offers valuable insights into the feasibility of achieving the goal of 100% renewable energy supply by 2050.

Keywords — *Electricity grid in Puerto Rico, two islands of Vieques and Culebra, distribution system operation, uncertainty, optimal power flow.*

NOMENCLATURE

A. Indices and Sets

t, i	Indices for time and electric buses
l	Indices electric lines
NT	Total time horizon
$\Phi_p(i), \Phi_c(i)$	Sets of parent and child buses for bus i

B. Parameters

SoC^{ini}	BESS initial state of charge
$\underline{SoC}, \overline{SoC}$	Minimum and maximum state of charge
η_c, η_d	Minimum and maximum BESS charging and discharging coefficients
$\overline{P}_i^{ch}, \overline{P}_i^{dis}$	Maximum BESS charging and discharging power
$\underline{P}_i^{ch}, \underline{P}_i^{dis}$	Minimum BESS charging and discharging power
λ_i	Electricity purchase price from the grid
$P_{i,t}^D, (Q_{i,t}^D)$	Active (reactive) load of the power distribution system
r_l, x_l	Resistance and reactance of line l
v^{\min}, v^{\max}	Minimum and maximum bounds for squared voltages.

$\overline{PV}_{i,t}$	Actual PV generation at bus i at time t
T_i^{on}	Minimum up time for charging or discharging operations of BESS at bus i

C. Variables

P_t^{sub}	Power supply from substation at time t
$p_{i,t}(q_{i,t})$	Active (reactive) power injection at bus i at time t
$PV_{i,t}$	PV dispatch at bus i at time t
$P_{i,t}^{ch}, P_{i,t}^{dis}$	Charging and discharging power of BESS at bus i at time t
$P_{l,t}(Q_{l,t})$	Active (reactive) branch flow on line l at time t
$v_{i,t}$	Square voltage at bus i at time t
$SoC_{i,t}$	BESS state of charge at bus i at time t
U_{it}^C	Binary variable for BESS charging mode: 1 if BESS charging and 0 otherwise.
U_{it}^D	Binary variable for BESS discharging mode: 1 if BESS discharging and 0 otherwise.

I. INTRODUCTION

Puerto Rico, with a population of about 3.2 million is an archipelago in the Caribbean Sea, primarily made up of one mainland island and two smaller island municipalities to the east as shown in Fig. 1. Its roughly rectangular shape is extended about 111 miles from east to west and 39 miles from north to south [1], [2].

Vieques, with a population of about 8,043 [3] and a land area of 55 square miles, which measures about 20 miles long by 4.5 miles wide, is an island and municipality of Puerto Rico, which lies about 8 miles east of Puerto Rico's mainland. The smaller island is Culebra, with a population of 1,769 [4] and a land area of 10 square miles, which measures about 7 miles long and 2 miles wide is Puerto Rico's least populous municipality. It is located approximately 17 miles east of Puerto Rico's mainland and 9 miles north of Vieques [5]-[7].

Two 38kV submarine cables connect Culebra and Vieques to the Luma's mainland power system (see Fig. 1), indicating that the grid reliability in Culebra and Vieques will depend solely on that of the mainland. In this context, non-wire

solutions, based on distributed energy resources (DERs) installed in Culebra and Vieques, could offer more promising alternatives than capital-intensive electricity infrastructure upgrades for maintaining the grid reliability in Culebra and Vieques. However, the amount of energy generated by photovoltaic (PV) systems is affected by weather conditions and solar irradiance. The uncertainty in the availability of PV generation poses major challenges in terms of balancing the supply with demand and affects the nodal voltage regulation and the distribution grid security.

One way of tackling these challenges is to use a very accurate PV generation forecasting strategy, which can help system operators anticipate the pertinent changes and take preventive measures, such as load shifting, volt-var control, and reactive power compensation, to maintain the system security and reliability. Indeed, the integration of battery energy storage system (BESS) with PV generation can provide a more flexible and controllable supply by reducing the impact of PV generation uncertainty. However, BESS alone would not suffice to address the challenges related to PV uncertainty as operational decisions would also need to be taken into consideration. The deterministic models for power distribution system operations assume that the input parameters are known with certainty, which can lead to non-optimal and expensive decisions in real-time operations due to the renewable energy resource variability and power system variations in real-time. As such, there is a need for stochastic modeling of operational schemes that can account for the uncertainty associated with practical cases.

The stochastic models incorporate a range of possible scenarios that reflect the prevailing uncertainties. These models provide more comprehensive analyses of power system operations and allow wider decision-making processes considering a range of possible scenarios. The scenario-based models provide a more accurate representation of system operations, allowing human operators to identify potential risks and vulnerabilities and take appropriate actions to mitigate such threats. Furthermore, stochastic models can be updated as new operating data becomes available, allowing system operators to continuously improve the accuracy of their models and optimize system performances over time.

In this context, this paper presents a stochastic approach for calculating the optimal power flows in Vieques and Culebra islands. We incorporate different options for representing the available solar PV generation, solve the optimal power flow problem using several operating scenarios, and discuss the impacts of PV generation uncertainty on the optimal operations of both islands.

The rest of the paper is organized as follows. Section II presents the mathematical formulation of the optimal power flow problem and discusses the proposed stochastic approach. The analysis of our different case studies is presented Section II.B. The conclusions are drawn in Section IV.

II. STOCHASTIC OPTIMAL POWER FLOW FOR DISTRIBUTION SYSTEM OPERATION

A. Deterministic optimal power flow problem

Considering the inherent nonlinear and nonconvex nature of the AC OPF problem, we use the linear *Distflow* OPF

formulation which is based on the branch flow model of the AC grid in order to establish a computationally viable distribution power flow model [9].

The OPF problem which is solved by the distribution system operator (DSO) will aim to minimize the cost of energy supplied by the distribution grid as shown in (1) with respect to operational constraints stated in (2)-(14):

$$\min \sum_t \lambda_t P_t^{sub} \quad (1)$$

$$p_{i,t} = -P_{i,t}^D + PV_{i,t} + (P_{i,t}^{dis} - P_{i,t}^{ch}); \quad q_{i,t} = -Q_{i,t}^D \quad (2)$$

$$\sum_{l \in \Phi p(i)} P_{l,t} - p_{i,t} = \sum_{l \in \Phi c(i)} P_{l,t} \quad (3)$$

$$\sum_{l \in \Phi p(i)} Q_{l,t} - q_{i,t} = \sum_{l \in \Phi c(i)} Q_{l,t} \quad (4)$$

$$v_{i,t} - v_{j,t} = 2(r_l P_{l,t} + x_l Q_{l,t}) \quad \forall l, \forall t \quad (5)$$

$$v^{\min} \leq v_{i,t} \leq v^{\max} \quad \forall i, \forall t \quad (6)$$

$$0 \leq PV_{i,t} \leq \overline{PV}_{i,t} \quad \forall i, \forall t \quad (7)$$

$$SoC_{i,t=1} = SoC^{ini}, \quad SoC_{i,t=NT} \geq \underline{SoC} \quad \forall i \quad (8)$$

$$SoC_{i,t} = SoC_{i,t-1} + (\eta_c P_{i,t}^{ch} - P_{i,t}^{dis} / \eta_d) \Delta t \quad \forall i, \forall t \quad (9)$$

$$U_{it}^C \overline{P_i^{ch}} \leq P_{i,t}^{ch} \leq U_{it}^C \underline{P_i^{ch}}; \quad \forall i, \forall t \quad (10)$$

$$U_{it}^D \overline{P_i^{dis}} \leq P_{i,t}^{dis} \leq U_{it}^D \underline{P_i^{dis}}; \quad \forall i, \forall t \quad (11)$$

$$0 \leq U_{it}^C + U_{it}^D \leq 1 \quad \forall i, \forall t \quad (11)$$

$$\underline{SoC} \leq SoC_{i,t} \leq \overline{SoC} \quad \forall i, \forall t \quad (12)$$

$$\sum_{t=t'}^{t'+T_i^{on}-1} U_{it}^C \geq T_i^{on} (U_{it}^C - U_{i(t-1)}^C) \quad (13)$$

$$t' = 1 \dots NT - T_i^{on} + 1; \quad \forall i$$

$$\sum_{t=t'}^{t'+T_i^{on}-1} U_{it}^D \geq T_i^{on} (U_{it}^D - U_{i(t-1)}^D) \quad (14)$$

$$t' = 1 \dots NT - T_i^{on} + 1; \quad \forall i$$

The nodal active and reactive power injection terms are expressed in (2). The active and reactive power balance are expressed in (3) and (4), respectively. The voltage across a given line l is given in (5) while voltage limits are stated in (6). The PV generation limits are given in (7). The BESS operational constraints are given in (8)-(14). The initial and final state of charge of BESS, at the end of the scheduling time horizon, are defined in (8). The BESS time-coupling energy balance constraint is presented in (9). Charging and discharging operations are restricted in (10) while (11) ensures that charging and discharging events are mutually exclusive. The range of operation for the BESS state of charge is given in (12). Charging and discharging events are constrained respectively to last for a minimum specified duration in (13) and (14) [10].

In summary the DSO's OPF problem subject to network constraints is presented as follows:

$$\min \sum_t \lambda_t P_t^{sub} \quad (15)$$

$$s.t \text{ (2)-(14)}$$



Fig. 1. Puerto Rico and Vieques and Culebra islands [8].

B. Proposed stochastic approach

Because of the inherently variable nature of PV generation, deterministic OPF models like (15) could fail to truly capture the real-time system operation characteristics. To palliate to this, we consider a stochastic approach that takes into account the realizations of multiple scenarios rather than solving the problem solely based on forecasted (deterministic) values. The proposed scenario-based approach is illustrated in Fig. 2. The four steps composing the proposed scenario-based approach are further detailed below.

Step 1: As we focus on the uncertainty that is stemming from PV generation, a set of possible N scenarios are first generated based on the Monte Carlo simulation method for modeling the uncertainties in the DSO's OPF problem. Using the Monte Carlo simulation method, a uniform weight p is assigned to each scenario to reflect the possibility of its occurrence.

Step 2: Because the computational performance of scenario-based models depends intrinsically on the number of scenarios, it becomes necessary to consider reduction techniques to make the final model less computationally intensive [11]. Consequently, the initial set of N scenarios is reduced to a subset of N_s preserved scenarios with a probability of occurrence of p_s for each scenario in the reduced subset. The General Algebraic Modeling System (GAMS) is used here for

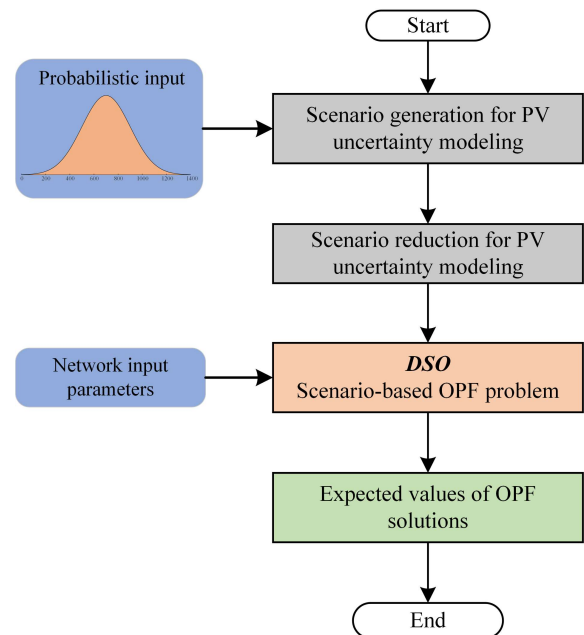


Fig. 2. Flowchart of the scenario-based approach.

the purpose of scenario reduction. More precisely, the SCENRED tool is used to reduce the initial set of generated PV output scenarios.

Step 3: Here, a revised formulation of (15) is used for the DSO's problem with the new objective function given in (16). Under the scenario-based approach, the objective of the DSO is to minimize the expected cost of energy supply from the grid across the subset of N_s preserved scenarios while guaranteeing that the operational constraints (2)-(14) are met for each scenario.

$$\min \sum_s \sum_t P_s \lambda_t P_{s,t}^{sub} \quad (16)$$

Step 4: Using the scenario-based OPF solution in Step 3, we obtain the expected values of the OPF solutions in terms of dispatch (i.e., PV dispatch, BESS schedules, and grid supply) and system operating conditions (i.e., voltage magnitudes and line flows).

III. CASE STUDIES

In this section, we provide an assessment of the impact of considering PV generation uncertainty on the distribution system operation in both Vieques and Culebra islands. We perform our simulations using the MATLAB 2022a platform with GUROBI solver on a laptop with an Intel Core i7-7700 CPU (3.6 GHz) and 16 GB of RAM.

A. Case Overview and Basic Settings

Simulations are performed for both Vieques and Culebra islands. The geovisualization information pertaining to the distribution network topology for Vieques and Culebra is displayed respectively in Fig. 3 and Fig. 4. As illustrated in Fig. 3, the Vieques' distribution network is composed of three 4.16 kV feeders differentiated by their color while the Culebra's distribution network is composed of two 4.16 kV feeders (see Fig. 4). Fig. 5 provides the graphical representation of some system parameters such as the system load, the PV generation profile, and the electricity price profile for both islands.



Fig. 3. Vieques geospatial distribution network.

Table I provides the load information at the feeder level for both islands. DERs such as PV and BESS are included in the presented distribution system with size information based on

ongoing efforts of LUMA to deploy microgrids on both islands. Table II provides the DER size information at the substation level. For the BESS operation, a 10% initial state of charge is considered for the allowable single daily cycle of charge/discharge with the minimum charging/discharging time of 2 hours. A total of 150 scenarios were initially generated based on the Monte Carlo simulation method, which are reduced to 5 scenarios using SCENRED, to execute our scenario-based OPF approach.



Fig. 4. Culebra geospatial distribution network.

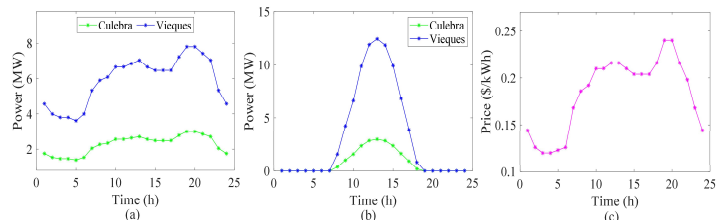


Fig. 5. Input parameters: (a) System load (b) Forecasted PV generation profile (c) Electricity price profile.

TABLE I
LOAD DEMAND AT FEEDER LEVEL FOR VIEQUES AND CULEBRA (MVA)

Location	Feeder 1	Feeder 2	Feeder 3
Vieques	2.642	3.373	1.842
Culebra	0.765	2.287	---

TABLE II
SYSTEM LEVEL DER INSTALLATIONS

Location	PV (MW)	BESS Power (MW)	BESS Capacity (MWh)
Vieques	12.5	7.7	7.7
Culebra	3	3.9	3.9

B. Simulation of the Aggregated System Operation

Here, the proposed scenario-based OPF approach is applied to an aggregated model for both islands. The aggregated model refers to the distribution system aggregation at the substation level, where only the generation constraints and the load demand are considered and the distribution network topology and network constraints (i.e., thermal loading and voltage limits) are disregarded. We consider the following two cases:

Case I: Deterministic OPF based on (15).

Case II: Proposed scenario-based OPF approach for considering the PV variability.

Fig. 6 illustrates the energy scheduling solution in Case I that come into play for feeder 2 in Culebra. Fig. 6 shows how generation resources are scheduled for minimizing the cost of supplying the load demand. We observe that BESS is charged at night when the grid price is at the lowest level and discharged at the peak evening hours when the grid price is the highest (see Fig. 5-c for prices). As the available PV generation increases throughout the day, we observe a steady decrease in the reliance on the main grid supply till late afternoon hours when the available PV power starts to diminish. At the peak of the PV generation (i.e., hours 12-15), we observe that there is no grid supply from the substation.

Fig. 7 presents the Cases I and II results for the aggregated simulation of Culebra. The results presented here focus on the solution for feeder 2 in Culebra. The results displayed in Fig. 7-a show that there is a smaller quantity of PV generation available in Case II, when considering the PV uncertainty, as compared to that in Case I, where only the forecasted values are considered. Having a smaller PV generation in Case II would lead to a higher grid supply from the substation as illustrated in Fig. 7-b. Furthermore, in Case I, there will be a higher PV generation output at midday, which would preclude the need for attaining any main grid supply at those hours. In contrast, the system must rely on the main grid supply even at PV peaking hours 12-15, which is due to a lower PV dispatch profile in Case II. The BESS energy charging/discharging schedule is shown in Fig. 7-c.

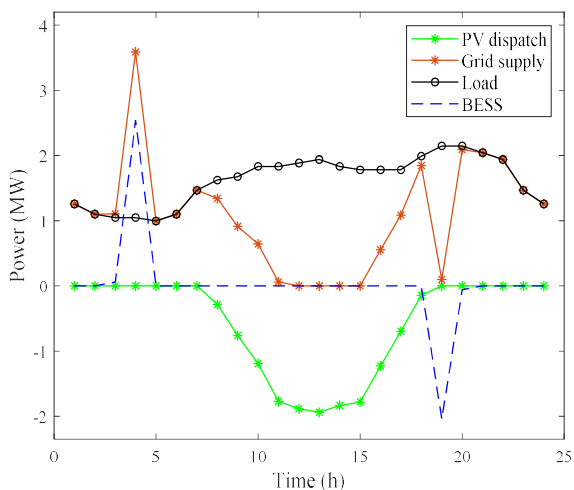


Fig. 6. Comprehensive results for feeder 2 in Culebra for Case I.

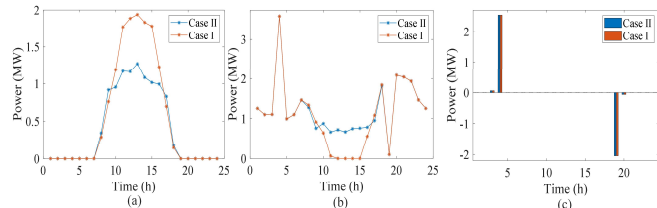


Fig. 7. Aggregated simulation for feeder 2 in Culebra: (a) Grid supply at substation (b) PV dispatch (c) BESS dispatch.

Similarly, Fig. 8 presents the Cases I and II results for the aggregated simulation of Vieques using feeder 2. Fig. 8-a shows that there is a clear PV dispatch discrepancy between Cases I and II. This difference would have an impact on the substation supply as illustrated in Fig. 8-b. Fig. 8-c shows that the BESS

dispatch strategy for Vieques differs from that of Culebra. Indeed, here, BESS is charged during day hours when grid prices are low. The reason is that the Vieques PV generation is higher than the load demand, which is used to charge the BESS located in Vieques. Here, the PV generation uncertainty has obviously affected the BESS charging in Vieques.

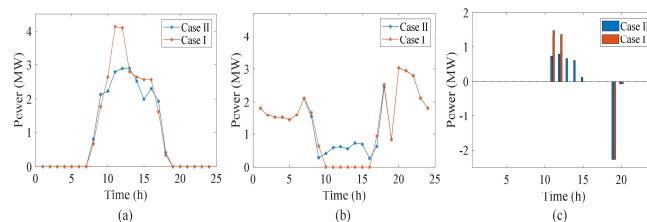


Fig. 8. Aggregated simulation for feeder 2 in Vieques: (a) Grid supply at substation (b) PV dispatch (c) BESS dispatch.

C. Simulation of Distributed System Operation

Here, the proposed scenario-based OPF approach is applied to a distributed model in both islands. The distributed model includes the constrained distribution network topology (i.e., thermal loading, voltage limits). Similar to the analyses presented in the previous section, we consider the following two cases for our study:

Case I: Deterministic OPF using (15).

Case II: Proposed scenario-based OPF approach for considering the PV variability.

Fig. 9 presents the Cases I and II results for the distributed simulation of the feeder 2 grid in Culebra. In addition to the clear differences between the deterministic Case I and the stochastic Case II, our simulation results point out additional differences in calculating the dispatch solutions in two cases. Under the deterministic Case I, we observe that the dispatched PV generation is lower than the available PV output, which results in a steady supply of power from the main grid even at peaking PV hours (i.e., see hours 12-15 in Fig. 9-b). This issue is essentially introduced by prevailing PV locations in the distribution grid, which mandate additional requirements for maintaining the nodal voltages within a prescribed range. The effect of the network constraints is also observed by the BESS operations, where the BESS charging occurs at midday instead of the off-peak evening hours as was seen in the aggregated simulation results.

Fig. 10 presents nodal voltage and line flow profiles for feeder 2 in Culebra under Case I. Fig. 10-a shows the expected network behavior such as steeper voltage drops during evening peak hours. We also observe the impact PV generation on the midday voltage profile, which has increased the nodal voltages that are adjacent to PV nodes. Fig. 10-b illustrates the line flows in feeder 2 with the plane surface representing the rated line capacity. As we move along the time-axis, we observe how the flow on main laterals changes following the load profile. The individual line flow profiles also show that the system is not congested even at the evening peak.

Fig. 11 presents the distributed simulation results for Vieques in Cases I and II. We observe how the uncertainty can influence the distributed line flow dispatch in Vieques. In contrast to the distributed line dispatch in Culebra, much of the available PV generation in Vieques is dispatched for reducing

the supply from the main grid under Case I, which is similar to that of the aggregated simulation (see Fig. 11-b). Fig. 12 presents nodal voltage profiles and line flows for feeder 2 in Vieques in the deterministic Case I. The displayed results confirm that the energy scheduling solution ensures that both bus voltages and line flows are within their respective prescribed range.

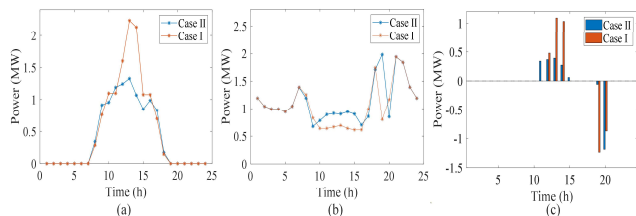


Fig. 9. Distributed simulation for feeder 2 in Culebra: (a) Grid supply at substation (b) PV dispatch (c) BESS dispatch.

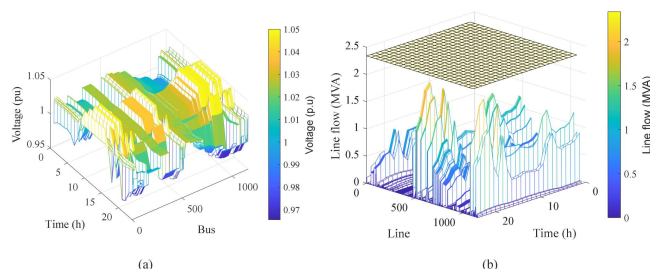


Fig. 10. Distributed simulation for feeder 2 in Culebra under Case I: (a) Voltage profiles (b) Line flows.

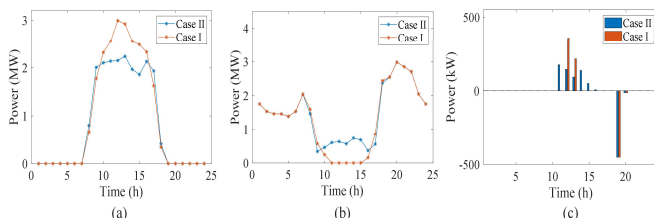


Fig. 11. Results for distributed simulation for feeder 2 in Vieques: (a) Grid supply at substation (b) PV dispatch (c) BESS dispatch.

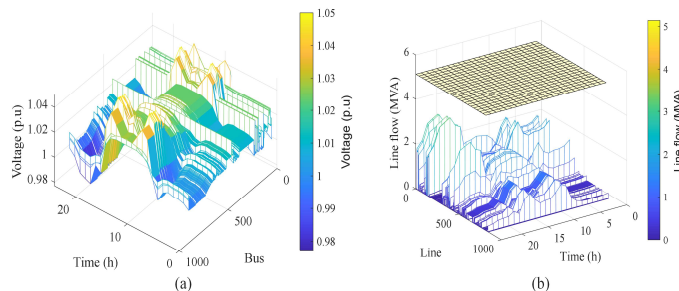


Fig. 12. Results for distributed simulation for feeder 2 in Vieques under Case I: (a) Individual voltage profiles (b) Individual line flows.

IV. CONCLUSION

Extreme and frequent weather events like hurricanes, culminating in large-scale power blackouts, continue to leave behind many unserved loads in Puerto Rico and its municipalities. To enhance the reliability and the resilience of the local grid while promoting clean energy objectives, renewable-based microgrids offer a promising solution. In our work, we have introduced a stochastic approach that highlights

the advantages of such methods in capturing the operating conditions of power systems. Unlike deterministic analyses, which rely on fixed inputs and predetermined scenarios, stochastic analyses consider the probabilistic nature of prevailing events, enabling a more comprehensive understanding of system behavior. By incorporating the PV variability into our model, we can better assess the impact of extreme weather events and their implications for enhancing the power supply resilience. Furthermore, our analyses have emphasized the importance of adopting more granular and detailed models rather than relying solely on aggregated system-based models at the substation level. The use of distributed models allows for a more accurate representation of the grid, capturing the specific characteristics and interactions at various nodes. Our simulation results have revealed significant discrepancies between aggregated and distributed models, substantiating the need for a more nuanced analysis that considers the unique attributes and complexities of the power system. By employing a stochastic approach and utilizing distributed, granular models, we can gain deeper insights into the operating conditions of the power system. This knowledge is invaluable in designing effective strategies to mitigate the impact of extreme weather events, reduce unserved loads, and ultimately improve the reliability and resilience of the grid while advancing clean energy goals in Puerto Rico and its municipalities.

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