Data Set for Manuscript "Day-ahead Market Clearing with Robust Security-Constrained Unit Commitment Model"

Hongxing Ye, Yinyin Ge, Mohammad Shahidehpour, Zuyi Li *

1 Data for Six-bus System

The one-line diagram is shown in Fig.1. The unit data and line data are shown in Table 1 and Table 2, respectively. Table 3 presents the load and uncertainty information. Column "Base Load" shows the hourly forecasted load. Assume that the load distributions are 20%, 40%, and 40% for Bus 3, Bus 4, and Bus 5, respectively. $\bar{u}_{1,t}$ and $\bar{u}_{3,t}$ in Table 3 are the bounds of the uncertainties at Bus 1 and Bus 3, respectively. The uncertainty bounds at other buses are 0.

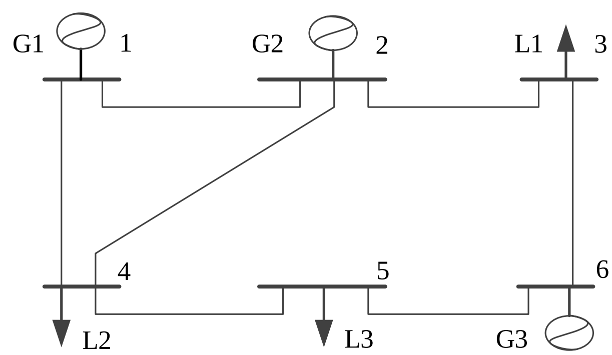

Figure 1: One-line Diagram for 6-bus system

Table 1: Unit Data for the 6-bus System

#	$P^{\min}$	$P^{\max}$	P_0	a	b	c	R^u	R^d	C_u	C_d	T^{on}	T^{off}	T_0
1	100	220	120	0.004	13.5	176.9	24	24	180	50	4	4	4
2	10	100	50	0.001	32.6	129.9	12	12	360	40	3	2	3
6	10	20	0	0.005	17.6	137.4	5	5	60	0	1	1	-2

$P^{\min}$,$P^{\max}$,P_0: min/max/initial generation level (MW);
fuel cost ($): $aP^2 + bP + c$;
R^u,R^d: ramping up/down rate (MW/h);
C_u,C_d: startup/shutdown cost ($);
T^{on},T^{off},T_0: min on/min off/initial time (h)

*This work is supported by the U.S. National Science Foundation Project Number ECCS-1549937. The authors are with the Robert W. Galvin Center for Electricity Innovation at Illinois Institute of Technology, Chicago, IL 60616, USA. (e-mail: hye9@hawk.iit.edu; yge9@hawk.iit.edu; ms@iit.edu; lizu@iit.edu).

Table 2: Line Data for the 6-bus System

from	1	1	2	5	3	2	4
to	2	4	4	6	6	3	5
x(p.u.)	0.17	0.258	0.197	0.14	0.018	0.037	0.037
capacity(MW)	200	100	100	100	100	200	200

Table 3: Load and Uncertainty Data for the 6-bus System (MW)

Time (h)	Base Load	$\bar{u}_{1,t}$	$\bar{u}_{3,t}$	Time (h)	Base Load	$\bar{u}_{1,t}$	$\bar{u}_{3,t}$
1	175.19	1.09	0.29	13	242.18	19.68	5.25
2	165.15	2.06	0.55	14	243.6	21.32	5.68
3	158.67	2.98	0.79	15	248.86	23.33	6.22
4	154.73	3.87	1.03	16	255.79	25.58	6.82
5	155.06	4.85	1.29	17	256	27.2	7.25
6	160.48	6.02	1.6	18	246.74	27.76	7.4
7	173.39	7.59	2.02	19	245.97	29.21	7.79
8	177.6	8.88	2.37	20	237.35	29.67	7.91
9	186.81	10.51	2.8	21	237.31	31.15	8.31
10	206.96	12.94	3.45	22	232.67	31.99	8.53
11	228.61	15.72	4.19	23	195.93	28.16	7.51
12	236.1	17.71	4.72	24	195.6	29.34	7.82

Table 4: Marginal Costs at Different Generation Levels ($/MWh)

Gen. 1			Gen. 2			Gen. 3		
$\underline{P}_1^w$	$\bar{P}_1^w$	mar. cost	$\underline{P}_2^w$	$\bar{P}_2^w$	mar. cost	$\underline{P}_3^w$	$\bar{P}_3^w$	mar. cost
100	124	14.396	10	28	32.638	10	12	17.71
124	148	14.588	28	46	32.674	12	14	17.73
148	172	14.78	46	64	32.71	14	16	17.75
172	196	14.972	64	82	32.746	16	18	17.77
196	220	15.164	82	100	32.782	18	20	17.79

2 Traditional SCUC Formulation

The objective of the ISOs/RTOs is to minimize the total operation cost to supply the load. It can be formulated as

$$\min \sum_i \sum_t C_i^P(P_{i,t}) + C_i^I(I_{i,t}), \tag{22a}$$

where $C_i^P(P_{it})$ is the fuel cost function in output level P_{it} for unit i, and $C_i^I(I_{it})$ is the cost function related to the unit status I_{it}. The objective function (22a) is subject to system generation-load balance constraint, as formulated in (22b), where the total generation equals the total load.

$$\sum_i P_{i,t} = \sum_m d_{m,t}, \forall t, \tag{22b}$$

where $d_{m,t}$ is the load demand at bus m at time t. The power loss is ignored in this paper. The transmission line flow constraint is modeled as

$$-F_l \leq \sum_m \Gamma_{l,m} \left(\sum_{i \in \mathcal{G}(m)} P_{i,t} - d_{m,t} \right) \leq F_l, \forall l, t. \tag{22c}$$

In the following context, we also denote the net power injection as $P^{\text{inj}}_{m,t}$.

$$P^{\text{inj}}_{m,t} := \sum_{i\in\mathcal{G}(m)} P_{i,t} - d_{m,t} \tag{23}$$

The power generation is subject to the unit capacity limits (24a), and unit ramping up/down limits (24b-24c).

$$I_{i,t}P_i^{\min} \leq P_{i,t} \leq I_{i,t}P_i^{\max}, \forall i, t \tag{24a}$$
$$P_{i,t} - P_{i,(t-1)} \leq r_i^u(1 - y_{i,t}) + P_i^{\min} y_{i,t}, \forall i, t \tag{24b}$$
$$-P_{i,t} + P_{i,(t-1)} \leq r_i^d(1 - z_{i,t}) + P_i^{\min} z_{i,t}, \forall i, t \tag{24c}$$

where $I_{i,t}$, $y_{i,t}$, and $z_{i,t}$ are the indicators of the unit being on, started-up, and shutdown, respectively. Units also respect the minimum on/off time constraints which are related to these binary variables [1].

3 Detailed Formulation for Problem (RSCUC)

$$\begin{aligned}
\text{(RSCUC)} \quad & \min_{(x,y,z,I,P)\in\mathcal{F}} \sum_t \sum_i \left(C_i^P(P_{i,t}) + C_i^I(I_{i,t})\right) \\
\text{s.t.} \quad & (22b), (22c), (24a) - (24c), \text{minimum on/off time limit}
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{F} :=\Big\{(x,y,z,I,P) : \forall \epsilon \in \mathcal{U}, \exists \Delta P \text{ such that} \\
& \sum_i \Delta P_{i,t} = \sum_m \epsilon_{m,t}, \forall t, , & (25a)\\
& I_{i,t}P_i^{\min} \leq P_{i,t} + \Delta P_{i,t} \leq I_{i,t}P_i^{\max}, \forall i, t & (25b)\\
& -R_i^d(1 - z_{i,t+1}) \leq \Delta P_{i,t} \leq R_i^u(1 - y_{i,t}), \forall i, t & (25c)\\
& \Delta P^{\text{inj}}_{m,t} = \sum_{i\in\mathcal{G}(m)} \Delta P_{i,t} - \epsilon_{m,t}, \forall m, t & (25d)\\
& -F_l \leq \sum_m \Gamma_{l,m}(P^{\text{inj}}_{m,t} + \Delta P^{\text{inj}}_{m,j}) \leq F_l, , \forall l, t\Big\}. & (25e)
\end{aligned}$$

The basic idea of the above model is to find a robust UC and dispatch for the base-case scenario. The UC and dispatch solution are immunized against any uncertainty $\boldsymbol{\epsilon} \in \mathcal{U}$. When uncertainty $\boldsymbol{\epsilon}$ occurs, it is accommodated by the generation adjustment $\Delta P_{i,t}$ (25a). Generation dispatch is also enforced by the capacity limits (25b). Equation (25c) models the ramping rate limits of generation adjustment $\Delta P_{i,t}$. In fact, the right and left hand sides of (25c) can correspond to a response time ΔT, which is similar to the 10-min or 30-min reserves in the literatures [2]. (25e) stands for the network constraint after accommodating the uncertainty.

4 Detailed Formulation for Problem (MP) and (SP)

$$\begin{aligned}
& \text{(MP)} \min_{(x,y,z,I,P,\Delta P)} \sum_t \sum_i \left(C_i^P(P_{i,t}) + C_i^I(I_{i,t})\right) \\
& \text{S.T.} \quad \text{(22b), (22c), (24a)-(24c), minimum on/off time limit} \\
& \sum_i \Delta P^k_{i,t} = \sum_m \epsilon^k_{m,t}, \forall t, \forall k \in \mathcal{K} & (26a)\\
& I_{i,t}P_i^{\min} \leq P_{i,t} + \Delta P^k_{i,t} \leq I_{i,t}P_i^{\max}, \forall i, t, \forall k \in \mathcal{K} & (26b)\\
& \Delta P^k_{i,t} \leq R_i^u(1 - y_{i,t}), \forall i, t, \forall k \in \mathcal{K} & (26c)
\end{aligned}$$

$$
\begin{aligned}
&-\Delta P_{i,t}^{k} \leq R_i^d(1 - z_{i,t+1}), \forall i, t, \forall k \in \mathcal{K} && \text{(26d)}\\
&-F_l \leq \sum_m \Gamma_{l,m}(P_{m,t}^{\text{inj}} + \Delta P_{m,t}^{\text{inj},k}) \leq F_l, \forall k \in \mathcal{K}, \forall l, t && \text{(26e)}\\
&\Delta P_{m,t}^{\text{inj},k} = \sum_{i \in \mathcal{G}(m)} \Delta P_{i,t}^{k} - \epsilon_{m,t}^{k}, \forall m, t, \forall k \in \mathcal{K}, && \text{(26f)}
\end{aligned}
$$

and

$$
\begin{aligned}
&\text{(SP) } \mathcal{Z} := \max_{\epsilon \in \mathcal{U}} \min_{(s^+, s^-, \Delta P) \in \mathcal{R}(\epsilon)} \sum_m \sum_t (s_{m,t}^+ + s_{m,t}^-) && \text{(27a)}\\
&\mathcal{R}(\epsilon) \quad := \Big\{(s^+, s^-, \Delta P): && \text{(27b)}\\
&\qquad \sum_i \Delta P_{i,t} = \sum_m (\epsilon_{m,t} + s_{m,t}^+ - s_{m,t}^-), \forall m, t && \text{(27c)}\\
&\qquad -F_l \leq \sum_m \Gamma_{l,m}\Big(P_{m,t}^{\text{inj}} + \Delta P_{m,t}^{\text{inj}}\Big) \leq F_l, \forall l, t && \text{(27d)}\\
&\qquad \Delta P_{m,t}^{\text{inj}} = \sum_{i \in \mathcal{G}(m)} \Delta P_{i,t} - (\epsilon_{m,t} + s_{m,t}^+ - s_{m,t}^-) && \text{(27e)}\\
&\qquad s_{m,t}^+, s_{m,t}^- \geq 0, \forall m, t && \text{(27f)}\\
&\qquad \text{(25b)}, \text{(25c)}\Big\}
\end{aligned}
$$

where $\mathcal{K}$ is the index set for uncertainty points $\hat{\boldsymbol{\epsilon}}$ which are dynamically generated in (SP) with iterations. It should be noted that $\hat{\boldsymbol{\epsilon}}^{\boldsymbol{k}}$ is the extreme point of $\mathcal{U}$. Variable $\Delta P_{i,t}^{k}$ is associated with $\hat{\boldsymbol{\epsilon}}^{\boldsymbol{k}}$. The objective function in (SP) is the summation of non-negative slack variables $s_{m,t}^+$ and $s_{m,t}^-$, which evaluates the violation associated with the solution (x, y, z, I, P) from (MP). $s_{m,t}^+$ and $s_{m,t}^-$ are also explained as un-followed uncertainties (generation or load shedding) due to system limitations.

5 Lagrangian Function for Problem (RSCED)

$$
\begin{aligned}
& \mathcal{L}(P, \Delta P, \lambda, \alpha, \beta, \eta) \\
= \; & \sum_t \sum_i C_i^P(P_{i,t}) + \sum_t \lambda_t \Big(\sum_m d_{m,t} - \sum_i P_{i,t}\Big) + \sum_t \sum_i \Big(\bar{\beta}_{i,t}(P_{i,t} - \hat{I}_{i,t} P_i^{\max}) + \underline{\beta}_{i,t}(\hat{I}_{i,t} P_i^{\min} - P_{i,t})\Big) \\
& + \sum_t \sum_i \Big(\bar{\alpha}_{i,t}\Big(P_{i,t} - P_{i,t-1} - r_i^u(1 - \hat{y}_{i,t}) - P_{i,t}^{\min}\hat{y}_{i,t}\Big) + \underline{\alpha}_{i,t}\Big(P_{i,t-1} - P_{i,t} - r_i^d(1 - \hat{z}_{i,t}) - P_{i,t}^{\min}\hat{y}_{i,t}\Big)\Big) \\
& + \sum_t \sum_l \Big(\bar{\eta}_{l,t}\Big(\sum_m \Gamma_{l,m} P_{m,t}^{\text{inj}} - F_l\Big) - \underline{\eta}_{l,t}\Big(\sum_m \Gamma_{l,m} P_{m,t}^{\text{inj}} + F_l\Big)\Big) + \sum_{k \in \mathcal{K}} \sum_t \lambda_t^k \Big(\sum_m \epsilon_{m,t}^k - \sum_i \Delta P_{i,t}^k\Big) \\
& + \sum_{k \in \mathcal{K}} \sum_t \sum_i \Big(\bar{\beta}_{i,t}^k(P_{i,t} + \Delta P_{i,t}^k - \hat{I}_{i,t} P_i^{\max}) + \underline{\beta}_{i,t}^k(\hat{I}_{i,t} P_i^{\min} - P_{i,t} - \Delta P_{i,t}^k)\Big) \\
& + \sum_{k \in \mathcal{K}} \sum_t \sum_i \Big(\bar{\alpha}_{i,t}^k\Big(\Delta P_{i,t}^k - R_i^u(1 - \hat{y}_{i,t})\Big) - \underline{\alpha}_{i,t}^k\Big(\Delta P_{i,t}^k + R_i^d(1 - \hat{z}_{i,t})\Big)\Big) \\
& + \sum_{k \in \mathcal{K}} \sum_t \sum_l \Big(\bar{\eta}_{l,t}^k\Big(\sum_m \Gamma_{l,m}(P_{m,t}^{\text{inj}} + \Delta P_{m,t}^{\text{inj},k}) - F_l\Big) - \underline{\eta}_{l,t}^k\Big(\sum_m \Gamma_{l,m}(P_{m,t}^{\text{inj}} + \Delta P_{m,t}^{\text{inj},k}) + F_l\Big)\Big) \qquad \text{(28)}
\end{aligned}
$$

6 Proofs for lemmas and theorems

6.1 Proof of Lemma 1

Proof. Consider $\hat{\epsilon}_{m,t}^k > 0, \pi_{m,t}^{\mathrm{u},k} < 0$. With a small perturbation $\delta > 0$ to $\hat{\epsilon}_{m,t}^k$, we replace $\hat{\epsilon}_{m,t}^k$ with $\hat{\epsilon}_{m,t}^k - \delta$ in (RSCED). As the $\pi_{m,t}^{u,k} < 0$, then the optimal value to problem (RSCED) increases. It means that there are violations for the original optimal solution $P_{i,t}$ to problem (RSCED) with $\hat{\epsilon}^k_{m,t} - \delta$. Hence, the optimal solution $P_{i,t}$ to problem (RSCED) cannot be immunized against the uncertainty $\hat{\epsilon}^k_{m,t} - \delta$. It contradicts with the robustness of the solution $P_{i,t}$. Therefore, if $\hat{\epsilon}^k_{m,t} > 0$, then $\pi^{\mathrm{u},k}_{m,t} \geq 0$. Similarly, if $\hat{\epsilon}^k_{m,t} < 0$, then $\pi^{\mathrm{u},k}_{m,t} \leq 0$. □

6.2 Proof of Lemma 2

Proof. Assume $i \in \mathcal{G}(m)$, according to the KKT condition

$$\frac{\partial \mathcal{L}(P, \Delta P, \lambda, \alpha, \beta, \eta)}{\partial \Delta P^k_{i,t}} = 0, \tag{29}$$

we have

$$\bar{\beta}^k_{i,t} - \underline{\beta}^k_{i,t} + \bar{\alpha}^k_{i,t} - \underline{\alpha}^k_{i,t} - \lambda^k_t + \sum_l (\bar{\eta}^k_{l,t} - \underline{\eta}^k_{l,t})\Gamma_{l,m} = 0. \tag{30}$$

Then (17) holds. If $\pi^{\mathrm{u},k}_{m,t} > 0$, then $\bar{\beta}^k_{i,t} + \bar{\alpha}^k_{i,t} > 0$ as $\bar{\beta}^k_{i,t}, \underline{\beta}^k_{i,t}, \bar{\alpha}^k_{i,t}$, and $\underline{\alpha}^k_{i,t}$ are non-negative. According to the complementary conditions for (7b) and (7d), at least one of (7b) and (7d) is binding. Hence, $\Delta P^k_{i,t} = \min\{\hat{I}_{i,t} P^{\max}_i - P_{i,t}, R^u_i (1 - \hat{y}_{i,t})\}$. Similarly, the other equation holds when $\pi^{\mathrm{u},k}_{m,t} < 0$. □

6.3 Proof of Theorem 1

Proof. The congestion fee at t is

$$\begin{aligned}
&\sum_m \Big(\pi^{\mathrm{e}}_{m,t} d_{m,t} - \sum_{i \in \mathcal{G}(m)} \pi^{\mathrm{e}}_{m,t} P_{i,t}\Big) = \sum_m \pi^{\mathrm{e}}_{m,t} P^{\mathrm{inj}}_{m,t} \\
=&\sum_m \sum_l \Gamma_{l,m} \Big(\bar{\eta}_{l,t} + \sum_{k \in \mathcal{K}} \bar{\eta}^k_{l,t} - \underline{\eta}_{l,t} - \sum_{k \in \mathcal{K}} \underline{\eta}^k_{l,t}\Big) P^{\mathrm{inj}}_{m,t} \\
=&\sum_l \Big(\bar{\eta}_{l,t} + \sum_{k \in \mathcal{K}} \bar{\eta}^k_{l,t} + \underline{\eta}_{l,t} + \sum_{k \in \mathcal{K}} \underline{\eta}^k_{l,t}\Big)(F_l - \Delta f_{l,t})
\end{aligned}$$

The first equality holds following the definition of net power injection. The second equality holds according to (8) and $\sum_m P^{\mathrm{inj}}_{m,t} = 0$. The third equality holds following (19). The sign change of $\underline{\eta}_{l,t}$ and $\sum_l \underline{\eta}^k_{l,t}$ in the third equation is because of the definition of power flow direction. The credits to FTR holders $\sum_{(m \to n)} (\pi^{\mathrm{e}}_{m,t} - \pi^{\mathrm{e}}_{n,t}) \mathrm{FTR}_{m \to n}$ can be rewritten as

$$\sum_{m \to n} \sum_l \left((\Gamma_{l,n} - \Gamma_{l,m}) \Big(\bar{\eta}_{l,t} - \underline{\eta}_{l,t} + \sum_{k \in \mathcal{K}} (\bar{\eta}^k_{l,t} - \underline{\eta}^k_{l,t})\Big) \right) \mathrm{FTR}_{m \to n}$$

Two cases are considered as follows.

1. If $\Delta f_{l,t}$ is non-zero, then $\bar{\eta}_{l,t}$ and $\underline{\eta}_{l,t}$ must be zeros. In this case, the congestion fee related to l at t is

$$\sum_{k \in \mathcal{K}} (\bar{\eta}^k_{l,t} + \underline{\eta}^k_{l,t})(F_l - \Delta f_{l,t}). \tag{31}$$

   And the credits to FTR holders are

$$\begin{aligned}
&\sum_{(m \to n)} (\pi^{\mathrm{e}}_{m,t} - \pi^{\mathrm{e}}_{n,t}) \mathrm{FTR}_{m \to n} \\
=&\sum_{(m \to n)} \sum_l \sum_{k \in \mathcal{K}} (\Gamma_{l,n} - \Gamma_{l,m})(\bar{\eta}^k_{l,t} - \underline{\eta}^k_{l,t}) \mathrm{FTR}_{m \to n} \\
\leq&\sum_l \sum_{k \in \mathcal{K}} (\bar{\eta}^k_{l,t} + \underline{\eta}^k_{l,t}) F_l
\end{aligned}$$

The first equality holds following $\bar{\eta}_{l,t} = \underline{\eta}_{l,t} = 0$. The inequality is true as the amount of $\text{FTR}_{m\to n}$ respects

$$-F_l \leq \sum_{m\to n} (\Gamma_{l,m} - \Gamma_{l,n})\text{FTR}_{m\to n} \leq F_l.$$

according to the SFT for FTR market [3–5]. Hence, the maximal credit to FTR holders is $\sum_{k\in\mathcal{K}}(\bar{\eta}^k_{l,t} + \underline{\eta}^k_{l,t})F_l$ for line l at t. Comparing with the congestion fee in (31), the FTR underfunding is (20).

2. If $\Delta f_{l,t}$ is zero, the congestion fee related to l at t is

$$(\bar{\eta}_{l,t} + \underline{\eta}_{l,t})F_l + \sum_{k\in\mathcal{K}}(\bar{\eta}^k_{l,t} + \underline{\eta}^k_{l,t})F_l,$$

which is the same as the maximal FTR credit. Hence, FTR underfunding is 0 at t for l.

□

6.4 Proof of Theorem 2

Proof. According to Theorem 1, the FTR underfunding value is (20). Therefore, we need to prove that the money collected from uncertainty sources can cover the FTR underfunding and credits to generation reserve.

Without loss of generality, we consider the payment collected from uncertainty sources at time t for $\hat{\boldsymbol{\epsilon}}^{\boldsymbol{k}}$

$$\begin{aligned}
&\sum_m \pi^{\text{u},k}_{m,t}\hat{\epsilon}^k_{m,t}\\
=&\sum_m \Big(\lambda^k_t - \sum_l \Gamma_{l,m}\left(\bar{\eta}^k_{l,t} - \underline{\eta}^k_{l,t}\right)\Big)\hat{\epsilon}^k_{m,t}\\
=&\sum_i \Delta P^k_{i,t}\lambda^k_t - \sum_m\sum_l \Gamma_{l,m}\left(\bar{\eta}^k_{l,t} - \underline{\eta}^k_{l,t}\right)\Big(\sum_{i\in\mathcal{G}(m)} \Delta P^k_{i,t}\Big)\\
&+\sum_l(\bar{\eta}^k_{l,t} + \underline{\eta}^k_{l,t})\Delta f_{l,t}\\
=&\sum_m\sum_{i\in\mathcal{G}(m)} \pi^{u,k}_{m,t}\Delta P^k_{i,t} + \sum_l(\bar{\eta}^k_{l,t} + \underline{\eta}^k_{l,t})\Delta f_{l,t}\\
=&\sum_i \pi^{u,k}_{m_i,t}\Delta P^k_{i,t} + \sum_l(\bar{\eta}^k_{l,t} + \underline{\eta}^k_{l,t})\Delta f_{l,t}
\end{aligned}$$

The first equality holds according to (9). According to (7a), (7f), and (7g), the $\sum_m\sum_l \Gamma_{l,m}\bar{\eta}^k_{l,t}\hat{\epsilon}^k_{m,t}$ in the second line can be rewritten as

$$\begin{aligned}
&\sum_m\sum_l \Gamma_{l,m}\bar{\eta}^k_{l,t}\Big(\sum_{i\in\mathcal{G}(m)}(\Delta P^k_{i,t} + P_{i,t}) - d_{m,t}\Big) - \sum_l \bar{\eta}^k_{l,t}F_l\\
=&\sum_m\sum_l \Gamma_{l,m}\bar{\eta}^k_{l,t}\sum_{i\in\mathcal{G}(m)}\Delta P^k_{i,t} + \sum_l \bar{\eta}^k_{l,t}\Big(\sum_m \Gamma_{l,m}P^{\text{inj}}_{m,t} - F_l\Big)\\
=&\sum_m\sum_l \Gamma_{l,m}\bar{\eta}^k_{l,t}\Big(\sum_{i\in\mathcal{G}(m)}\Delta P^k_{i,t}\Big) + \sum_l \bar{\eta}^k_{l,t}\Delta f_{l,t}
\end{aligned}$$

Hence, the second equality holds. The third equality holds from (9). Therefore,

$$\sum_m\sum_t \Psi_{m,t} = \sum_i\sum_t \Theta^G_{i,t} + \sum_l\sum_t \Theta^T_{l,t},$$

the payment collected from uncertainty sources can cover all the credits to the generation and transmission reserves. □

6.5 Proof of Competitive Equilibrium

Proof. $P_{i,t}$ and $(Q_{i,t}^{\text{up}}, Q_{i,t}^{\text{down}})$ are coupled by constraints (15) and (16). According to (17), we can rewrite generation reserve credit as

$$\begin{aligned}
\pi_{m,t}^{\text{u,up}} Q_{i,t}^{\text{up}} + \pi_{m,t}^{\text{u,down}} Q_{i,t}^{\text{down}} &= \sum_{k\in\mathcal{K}} \pi_{m,t}^{\text{u},k} \Delta P_{i,t}^{k} \\
=& \sum_{k\in\mathcal{K}} (\bar{\beta}_{i,t}^{k} - \underline{\beta}_{i,t}^{k} + \bar{\alpha}_{i,t}^{k} - \underline{\alpha}_{i,t}^{k}) \Delta P_{i,t}^{k} \\
=& \sum_{k\in\mathcal{K}} \begin{pmatrix} \bar{\beta}_{i,t}^{k}(\hat{I}_{i,t} P_i^{\max} - P_{i,t}) + \underline{\beta}_{i,t}^{k}(P_{i,t} - \hat{I}_{i,t} P_i^{\min}) \\ +\bar{\alpha}_{i,t}^{k}(R_i^{u}(1-\hat{y}_{i,t})) + \underline{\alpha}_{i,t}^{k} R_i^{d}(1-\hat{z}_{i,t+1}) \end{pmatrix}
\end{aligned} \tag{32}$$

Substituting (32) into problem (PMP$_i$), we can decouple $P_{i,t}$ and $(Q_{i,t}^{\text{up}}, Q_{i,t}^{\text{down}})$. In fact, we also get all terms related to $P_{i,t}$ in Lagrangian $\mathcal{L}(P, \lambda, \alpha, \beta, \eta)$ for problem (RSCED). Since the problem (RSCED) is linear programming problem. Therefore, the saddle point $\hat{P}_{i,t}$, which is the optimal solution to (RSCED), is also the optimal solution to (PMP$_i$). Consequently, unit i is not inclined to change its generation output level as it can obtain the maximum profit by following the ISO's dispatch instruction $\hat{P}_{i,t}$. Therefore, dispatch signal $\hat{P}_{i,t}$ and price signal $(\pi_{m,t}^{\text{e}}, \pi_{m,t}^{\text{u},k})$ constitute a competitive partial equilibrium [6]. $\square$

7 Stoarge Model in Case 2 for IEEE 118-Bus

$$\begin{aligned}
& E_t = E_{t-1} + \rho^d P_t^D + \rho^c P_t^C, \forall t \\
& 0 \le E_t \le E^{\max}, \forall t \\
& 0 \le -P_t^D \le I_t^D R^D, \forall t \\
& 0 \le P_t^C \le I_t^C R^C, \forall t \\
& I_t^D + I_t^C \le 1, \forall t \\
& E_{N_T} = E_0,
\end{aligned}$$

where E_t denotes the energy level, P_t^D and P_t^C represent the discharging and charging rates, and I_t^D and I_t^C are the indicators of discharging and charging. As the UMP is the major concern in this section, we use simplified parameters for storage. The discharging efficiency ρ^d and charging efficiency ρ^c are set to 100%. The capacity $E^{\max}$ and initial energy level E_0 are set to 30 MWh and 15 MWh, respectively. The maximal charging rate R^D and discharging rate R^C are set to 8 MW/h.

References

[1] M. Shahidehpour, H. Yamin, and Z. Li, *Market Operations in Electric Power Systems: Forecasting, Scheduling, and Risk Management*, 1st ed. Wiley-IEEE Press, 2002.

[2] Z. Li and M. Shahidehpour, "Security-constrained unit commitment for simultaneous clearing of energy and ancillary services markets," *IEEE Trans. Power Sys.*, vol. 20, no. 2, pp. 1079–1088, 2005.

[3] W. W. Hogan, "Financial transmission right formulations."

[4] "Pjm manual on financial transmission rights," PJM, Tech. Rep.

[5] W. W. Hogan, "Contract networks for electric power transmission," *Journal of Regulatory Economics*, vol. 4, no. 3, pp. 211–242, 1992.

[6] A. Mas-Colell, M. D. Whinston, J. R. Green *et al.*, *Microeconomic theory.* Oxford university press New York, 1995, vol. 1.