# Appendix A

Here we present a new way to determine the uncertainty budget according to a specified confidence level. The polyhedral representation for the uncertainty is given as

 (A1)

Let . If  is a sequence of independent and identically distributed random variables, then  is also a sequence of independent and identically distributed random variables. We further assume and . According to the *Lindeberg–Lévy central limit theorem*, we have

 (A2)

If the confidence probability for the budget constraints is set as , we get

 (A3)

Then, it attains

 (A4)

Note that although the number of time slots *T* is 24 in our work, it is assumed that the random variable approximatively converges in distribution to a normal *N* (0,1). When the number of time slots increases, for example to 96 (one day is divided into 96-time intervals and each interval represents 15 minutes), the accuracy increases accordingly. The detailed study on how the number of time slots influences the convergence of the random variable toward a normal distribution is out of scope of this paper.

Next, we present an example for deriving the expectation  and variance  of random variable . It is assumed that the wholesale electricity price  is distributed normally with  and . Additionally, we assume so that the estimate interval  can capture the true value of  with a confidence level of 99.74% . On this basis, we have

 and  (A5)

For brevity, the subscript *t* is neglected hereinafter. Let denote the probability density function with respect to random variable . We have



 (A6)

 (A7)

When the confidence probability for the budget constraints is set as 99.99%, the lower limit for the budget is 7.17.

# Appendix B

Here, we present a detailed proof based on the method proposed in [A1]. The utility function in our model is linearized by a piecewise-linear function. As a large number of demand response resources are aggregated, we approximately assume the power of each DRA  is a continuous variable.

For brevity, we define and . We define  and the pseudo gradient of  can be defined as follows:

 (B1)

Definition:  is diagonally strictly concave for **x** if for every **x**1, **x**0, we have

**** (B2)

where  denotes the transpose of **x**.

Next, we show is diagonally strictly concave. According to (B1), we get

 (B3)

where 

Define:

 (B4)

 (B5)

Substituting (B4) and (B5) into (B2), as and  , we have,

 (B6)

Thus, is diagonally strictly concave. In addition, is a concave function (In our model it is a concave quadratic function) and the allowed strategies are limited by the requirement that the decision variables (e.g., ) is selected from a convex and closed set (In our model it is a closed polyhedral set as represented by  ). According to Theorems 1 and 2 in [A1], a Nash equilibrium always exists and is unique.

[A1] J. B. Rosen, “Existence and uniqueness of equilibrium points for concave n-person games,” *Econometrica*, vol. 33, pp. 347–351, 1965.

# Appendix C

Here we show how to determine the lower limit of *M* in (42). The Lagrangian multiplier  is the sensitivity of the objective function (20) of DRA with respect to the right-hand term . Suppose that  increases to , the optimal value of objective function of DRA *d* is also changed accordingly. First, let us look into the change in the cost term. The maximal incremental cost may happen at the highest retail price period. Thus, the incremental cost respects the following inequality:



 (C1) The maximal change in the utility function is described as

 (C2)

Let denote the change in the objective function. Both and increase monotonically with respect to. Therefore, and . We have

 (C3)

As a result, we get

 (C4)

We assume DRA has the same utility function for every time interval. If not, we enumerate all time intervals and choose the largest one to substitute into (C4).

The above is seen as a general method to determine an appropriate value of *M*1. Besides, in our model as we can determine the sign of  and in advance, we derive a tighter lower limit on *M*1. As  increases to , we have . Thus, it is only necessary to calculate the minimal value of .Thus,

If 

Then

 (C5)

Otherwise, according to and , we have

 and  (C6)

In this case, . This is reasonable because  equal to zero implies that the inequality (42) is not binding and thus the corresponding Lagrangian multiplier is zero. In our model, *M* is set at 120.