

Risk-Constrained Bidding Strategy with Stochastic Unit Commitment

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Abstract—This paper develops optimal bidding strategies based on hourly unit commitment in a generation company (GENCO) which participates in energy and ancillary services markets. The price-based unit commitment problem with uncertain market prices is modeled as a stochastic mixed integer linear program. The market price uncertainty is modeled using the scenario approach, Monte Carlo simulation is applied to generate scenarios, scenario reduction techniques are applied to reduce the size of the stochastic price-based unit commitment problem, and postprocessing is applied based on marginal cost of committed units to refine bidding curves. The financial risk associated with market price uncertainty is modeled using expected downside risk which is incorporated explicitly as a constraint in the problem. Accordingly, the proposed method provides a closed-loop solution to devising specific strategies for risk-based bidding in a GENCO. Illustrative examples show the impact of market price uncertainty on GENCO's hourly commitment schedule and discuss the way GENCOs could decrease financial risks by managing expected payoffs.

Index Terms— Risk, bidding strategy, stochastic price-based unit commitment, mixed integer programming

I. NOMENCLATURE

Variables:

$EDR()$	Expected downside risk for a given target profit
i	Denote a thermal unit
$I()$	Unit status indicator with 1 means on and 0 means off
$I_d()$	Indicator for providing non-spinning reserve when off
$F()$	Unit's consumption of fuel for a scenario
m	Segment index
n	Denote a node in the scenario tree
n'	Denote a node different from node n in the scenario tree
$N_u()$	Non-spinning reserve of a unit when on
$N_d()$	Non-spinning reserve of a unit when off
$p_m()$	Generation of segment m in linearized heat curve
$P()$	Generation of a unit
$PF()$	Payoff for a scenario
$R()$	Spinning reserve of a unit
$RISK()$	Downside risk for a scenario

s	Denote a scenario
t	Hour index
$TP()$	Total generation offered to (positive value) or purchased from (negative value) the market by unit
a	
$TN()$	Total non-spinning reserve offered by a unit
$v_m()$	Indicate whether a unit is started at segment m of the startup cost curve, 1 means started at segment m and 0 means off
$x()$	Auxiliary binary variable for one scenario
$z()$	Shutdown indicator

Constants:

$b_m()$	Slope of segment m in linearized heat curve
$f()$	Heat rate at the minimum generating capacity
$\rho_f()$	Fuel price of a unit
$HN()$	Set of nodes for an hour
$MSR()$	Maximum sustained ramp rate (MW/min) for a unit
$NS()$	Number of segments for the startup fuel curve
$NSF()$	Number of segments for the piece-wise linearized heat rate curve
$P_0()$	Bilateral contract of a unit
$SG_0()$	Income from bilateral contract of a unit
$\underline{P}_g(), \bar{P}_g()$	Minimum/maximum generating capacity
$QSC()$	Quick start capacity
$RU(), RD()$	Ramping up/down limit of a unit
$SD()$	Shutdown cost of a unit
$SF_m()$	Startup fuel if started at segment m
$SN()$	Set of nodes for a scenario
z_0	Targeted profit
$\pi()$	Probability for a scenario
$\rho_g()$	Market price for energy
$\rho_{sr}()$	Market price for spinning reserve
$\rho_{nr}()$	Market price for non-spinning reserve

II. INTRODUCTION

DEVELOPING a good bidding strategy is very crucial for a GENCO to maximize its potential profit [1,2]. The approaches for developing bidding strategies could be categorized into: equilibrium models and non-equilibrium

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models. Equilibrium models such as Supply Function Equilibrium and Cournot Equilibrium were widely applied for developing GENCOs' bidding strategies and analyzing market power in energy markets [2]-[5]. However, unit constraints such as minimum on/off time, ramping limits, and startup cost were not considered in most of the equilibrium models because the existence of equilibria could not be proven when integer variables were used in those models. Accordingly, the simulated market equilibrium without the unit prevailing constraints could deviate largely from practical operation. Meanwhile, there may be some computational problems when equilibrium models are applied to a large system with many market participants. However, equilibrium models would be very important for analyzing the potential market power of a GENCO and the optimal bidding strategy of GENCOs with market power.

There are several non-equilibrium approaches in the literature for developing optimal bidding strategies. For example, an ordinal optimization method was used in [6] to find the "good enough" bidding strategy for power suppliers. The basic idea was to use an approximate model for analyzing the impact of GENCO's bidding strategies on market clearing price. A bidding model was proposed in [7] based on an economic principle known as cobweb theorem. The proposed model calculated the maximum bidding price and quantity by an iterative procedure using the GENCO's residual demand curve. Deterministic price-based unit commitment (PBUC) was applied for developing bidding strategies in [1], [8]-[11]. However, the precision of market price forecasting could have a direct impact on PBUC solution. Due to electricity market dynamics, which could make it difficult to forecast market prices accurately, it would be very important to consider the market price uncertainty.

There are several approaches to modeling the market price uncertainty. The first approach is to model directly the market price uncertainty. Without considering ancillary services, the stochastic unit commitment problem in a PoolCo market with uncertain market prices was solved using LR, stochastic dynamic programming, and Benders decomposition in [12]. However, the purpose of [12] was to develop policies including unit commitment and generation dispatch for each scenario instead of developing bidding strategies. The market price uncertainty was taken into consideration in [13] when developing optimal bidding strategies in multi-markets. The second approach is to model the uncertainty of residual demand curve. The uncertainty of residual demand curve was considered in [14] by applying the scenario approach to develop optimal bidding strategies for a GENCO. Benders decomposition was employed to solve the corresponding stochastic linear program. The third approach is to model the uncertainty of competitors' bids and system loads. The uncertainty of competitors' bids was modeled in [15] for developing optimal bidding strategies in energy market. Binary expansion approach was applied to transform the bi-level optimization problem into a mixed integer linear program, and the resulting problem was solved by an MIP solver. In [13] and [14], unit commitment decisions were not

considered and assumed to be given. Meanwhile, models in [13]-[15] were risk-neutral in the sense that the objective was to maximize expected payoffs. The optimal decision could expose the GENCO at a significant risk level because of the market price uncertainty. It would be very important for a GENCO to maximize its potential profit while keeping the involved risk at an acceptable level.

There are several ways to model risks associated with a decision. A very common way is the mean-variance approach proposed in [16], which is the industry's standard model in portfolio selection. In this approach, the risk is measured using the variance of the expected payoff and a utility function is devised by appending the variance of expected payoff into the original expected payoff function. The objective of a decision maker would be to maximize its utility function. In the case of stochastic integer program, this method is criticized for its computational intractability when appending a variance term in the objective function [17]. The value at risk (VaR) approach [1] was applied to analyze risk associated with a bidding strategy. However, an open-loop solution made it very difficult for a GENCO to modify its bidding strategy based on the risk level. Real option models and stochastic optimization techniques were applied in [18] to manage risk. A detailed overview of risk assessment method in energy trading was given in [19].

This paper proposes a risk-constrained bidding strategy for a GENCO to devise optimal bids in day-ahead energy market and ancillary services market. Our proposed method belongs to the non-equilibrium approach category. The problem is formulated as a stochastic mixed integer linear programming and solved by commercial mixed integer programming (MIP) solver. The tradeoff between maximizing expected payoff and minimizing risk due to the market price uncertainty is modeled explicitly by including the expected downside risk as a constraint. Accordingly, the proposed procedure provides a closed-loop solution to devising bidding strategy. Meanwhile, prevailing unit commitment constraints are considered. After solving the stochastic PBUC problem, postprocessing techniques based on marginal cost are applied to refine the bidding curves. Illustrative examples show the impact of market price uncertainty on commitment schedule of generators and a GENCO could significantly decrease the level of involved risk at the cost of reducing its expected payoff.

This paper is organized as follows: the stochastic PBUC problem is formulated in section III and the construction of bidding curve is shown in section IV. Section V models risk constraint. Illustrative examples and conclusions are provided in sections VI and VII, respectively.

III. STOCHASTIC PRICE-BASED UNIT COMMITMENT

We devise bidding strategies simultaneously for energy and ancillary services markets. If ancillary services are cleared after the day-ahead energy market, we would apply the proposed method to submit energy bidding curves and execute the program again by plugging in day-ahead energy results. In

this way, the certainty of market clearing results in the day-ahead energy market could enhance a GENCO's profit.

Market prices could be stated as locational marginal prices (LMPs) or uniform market clearing prices (MCPs) which depend on market rules in consideration. Our model could be applied to either case. As market price variables are represented by continuous probability distributions, it is very difficult, if not impossible, to solve the corresponding stochastic programming problem since integration over such variables are required explicitly or implicitly. To overcome this problem, we generally resort to approaches that could substitute the continuous market price variables with a set of discrete outcomes. Each possible discrete outcome of market price is called a scenario. We discuss next the approach for generating market price scenarios based on forecasted market prices.

A. Scenario generation

There are various approaches to generating scenarios for stochastic programming. Scenarios are commonly generated by sampling historical time series or statistical models such as time series or regression models. Time series models were applied to generate scenarios for prices in electricity markets in [13]. A nonlinear optimization problem was employed in [20] to generate scenario trees given the statistical properties of stochastic variables. A detailed literature review on scenario generation was presented in [21]. In this paper, the Monte Carlo simulation method is applied to generate scenarios. Market price and market price variance forecasts for energy and ancillary services are assumed to be calculated by applying techniques such as time series and artificial neural network [1]. Then, Monte Carlo simulation is executed M (i.e., a very large number) times to generate scenarios for market prices when the probability of each scenario is assumed to be $1/M$.

B. Scenario reduction

If we execute Monte Carlo simulation 100,000 times, we would obtain 100,000 scenarios and the resulting stochastic program would be too large to solve. Accordingly, scenario reduction techniques are applied to reduce the number of scenarios in consideration while maintaining a good approximation of the statistical properties of market prices. The basic idea of scenario reduction is to eliminate a scenario with very low probability and bundle scenarios that are very close. Accordingly, scenario reduction algorithms determine a subset of scenarios and calculate probabilities for new scenarios such that the reduced probability measure is closest to the original probability measure in terms of a certain probability distance between the two measures [22], [23].

After reducing and bundling scenarios, an example of the reduced scenario tree for a three-stage problem is shown in Fig.1.

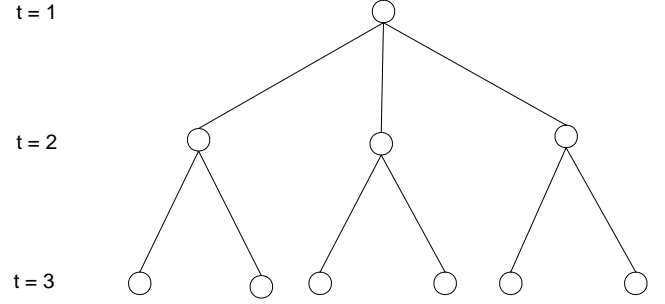


Fig. 1 Scenario tree for a three-stage stochastic problem

A node in the scenario could have multiple successors but one ancestor at most. The node without any ancestor is called the root node and nodes without any successors are called leaves. A scenario is defined as a path in the scenario tree from the root node to a leaf node. For a node n in the scenario tree, we apply n_+ to denote its predecessor. Meanwhile, we apply n_0 to denote the root node in the scenario tree.

Since a small number of scenarios could result in a poor approximation, a tradeoff exists between problem precision and problem size. A practical way is to set the number of reduced scenarios when the objective function is not changing significantly or the relative distance between original scenarios and reduced scenarios is within an acceptable level [22], [23].

C. Objective function for a GENCO

A GENCO intends to maximize its expected payoff as:

$$\max_s : \sum_s \pi(s) * PF(s) \quad (1)$$

and the profit for scenario s is:

$$PF(s) = \sum_i \left\{ \begin{array}{l} -\rho_f(i) * F(i, s) \\ + \sum_{n \in SN(s)} \left\{ \begin{array}{l} \rho_g(i, n) * TP(i, n) + \rho_{sr}(i, n) * R(i, n) \\ + \rho_{nr}(i, n) [N_u(i, n) + N_d(i, n)] \\ + SG_0(i, n) - z(i, n) * SD(i) \end{array} \right\} \end{array} \right\} \quad (2)$$

Since all the information (i.e., unit status, price, generation, etc.) for hour t is included in the set of nodes $HN(t)$ at that hour, we present the equations as a function of node n instead of hour t . We present the startup cost as a function of startup fuel (MBtu), for modeling the consumption of constrained fuel, and shutdown cost as function of cost (dollars). However, we could also present the startup cost as a function of cost (dollars) instead of fuel (MBtu). The variation would not impact the proposed formulation.

In this paper, we consider thermal units. However, other types of units such as combined-cycle, cascaded-hydro, and pumped-storage units could be included without much difficulty [11]. We discuss the various constraints for a GENCO as follows.

D. Unit constraints

A GENCO would make unit commitment decisions before submitting bids to the day-ahead market. Accordingly, the final unit commitment decision should be the same for all possible scenarios of market price. If a system operator, i.e.,

an ISO or RTO, is responsible for commitment decisions, decisions could be different from one scenario to another. The unit-specific constraints were given in [11] such as minimum on/off time limits and time-varying startup costs. If the outcome of stochastic variables for a certain number of scenarios is the same, decision variables for those scenarios should also be the same. This is called nonanticipativity constraints [24]. For a stochastic program, there are two ways of expressing nonanticipativity constraints: scenario-based and node-based. In the scenario-based approach, nonanticipativity constraints are enforced explicitly [13] which introduce additional constraints and variables to the problem. In the node-based approach, constraints are formulated for each node instead of each scenario and nonanticipativity constraints are enforced implicitly as shown next.

Ramping constraints:

$$\begin{aligned} P(i, n) - P(i, n_+) &\leq RU(i) && \forall n \\ P(i, n_+) - P(i, n) &\leq RD(i) && \forall n \end{aligned} \quad (3)$$

Energy bilateral contracts for units:

$$P(i, n) - TP(i, n) = P_0(i, n) \quad \forall i, n \quad (4)$$

where positive $TP(i, n)$ represents the generation offered to the market and negative $TP(i, n)$ is the purchased generation from the market.

The market price for a higher quality reserve (spinning reserve) should be higher than that of a lower quality reserve (non-spinning reserve). However, this is not always observed in practical markets because of market inefficiencies. Accordingly, a general case of generating unit constraints for supplying energy and ancillary services in spot market is modeled in (5):

$$\begin{aligned} P(i, n) + R(i, n) + N_u(i, n) &\leq \bar{P}_g(i) * I(i, n) && \forall i, n \\ \underline{P}_g(i) * I(i, n) &\leq P(i, n) && \forall i, n \\ R(i, n) + N_u(i, n) &\leq 10 * MSR(i) * I(i, n) && \forall i, n \\ N_d(i, n) &\leq QSC(i) * I_d(i, n) && \forall i, n \\ I_d(i, n) + I(i, n) &\leq 1 && \forall i, n \end{aligned} \quad (5)$$

The fuel consumption of unit i for scenario s is calculated as:

$$\begin{aligned} F(i, s) &= \sum_{n \in SN(s)} [(f(i)I(i, n) + \sum_{m=1}^{NSF(i)} p_m(i, n)b_m(i)) + \sum_{m=1}^{NS(i)} v_m(i, n)SF_m(i)] \\ P(i, n) &= \underline{P}_g(i)I(i, n) + \sum_{m=1}^{NSF(i)} p_m(i, n) && \forall i, n \\ 0 \leq p_m(i, n) &\leq \bar{p}_m(i) && \forall i, n \end{aligned} \quad (6)$$

Other constraints could be modeled similarly as shown in [11].

IV. CONSTRUCTION OF BIDDING CURVE

Ideally, a market price in real time should correspond to one scenario for the realization of possible market prices. In this sense, the generation obtained in section III is the optimal bidding quantity for a GENCO at the corresponding market price. Accordingly, price and generation pairs obtained in section III could be applied to construct a bidding curve [14],

[15]. In practical electricity markets, a market participant would submit a piecewise non-decreasing bidding curve such as that in Fig. 2.

A. Non-decreasing conditions

In the stochastic integer program formulated in section III, scenarios are treated independently. However, the hourly price and generation quantity pairs may not be monotonically increasing. Accordingly, the non-decreasing conditions are enforced as:

$$\begin{aligned} [\rho_g(n) - \rho_g(n')] * [TP(i, n) - TP(i, n')] &\geq 0 \quad \forall n, n' \in HN(t), \forall i, t \\ [\rho_{sr}(n) - \rho_{sr}(n')] * [R(i, n) - R(i, n')] &\geq 0 \quad \forall n, n' \in HN(t), \forall i, t \\ [\rho_{nr}(n) - \rho_{nr}(n')] * [TN(i, n) - TN(i, n')] &\geq 0 \quad \forall n, n' \in HN(t), \forall i, t \end{aligned} \quad (7)$$

A GENCO is bound to enforce non-decreasing conditions for its total generation, total spinning reserves, and total non-spinning reserves if it submits bidding curves for the entire GENCO instead of its individual units.

B. Construction of bidding curve

The problem solution presents hourly non-decreasing price and quantity pairs. A sample for a unit is shown in Fig. 3 in which there is a large jump from the second pair to the third. In such cases, there are two possible ways of constructing a continuous bidding curve as shown in Fig. 4.

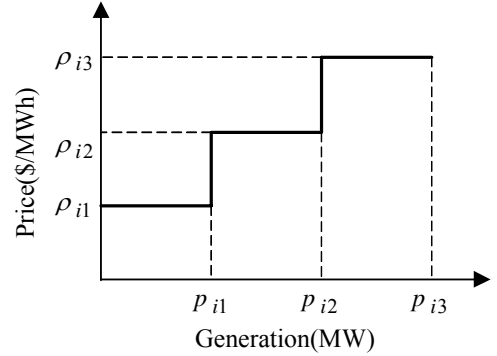


Fig.2. Bidding curve for unit i

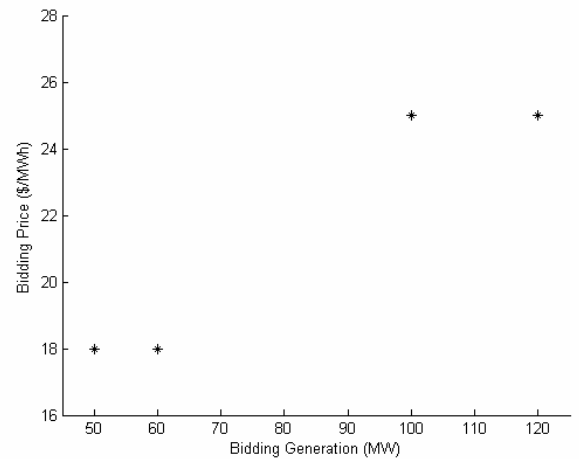


Fig.3. Bidding price and quantity pairs

The first method is to use the lower generation and upper price of two continuous points to obtain a new price and quantity pair and connect all the price and quantity pairs to devise the bidding curve. By applying this method, the unit may lose some revenue if the market price is between \$18/MWh and \$25/MWh since the unit will only be awarded 60MW.

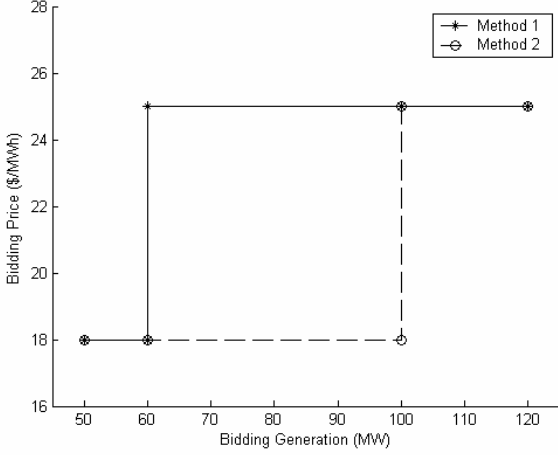


Fig.4. Two natural methods for constructing bidding curve

This is a conservative way of constructing bidding curves. The second method is to use the upper generation and lower price of two continuous points to obtain new price and quantity pairs. This would be a risky method as a unit may incur losses if the market price is between \$18/MWh and \$25/MWh and the awarded generation is 100MW.

Based on the two methods, we propose two other ways to construct bidding curves. After finding price and quantity pairs, the unit status, no-load cost, and startup/showdown cost are determined. The marginal cost of a GENCO would determine whether or not the GENCO should offer additional generation. Accordingly, marginal cost could be utilized for constructing bidding curves.

Assume we have two continuous price and quantity pairs m' and m . If the generation difference between the two continuous price and quantity pairs, $\Delta P(i, m) = P(i, m') - P(i, m)$, is larger than a given tolerance ε_g and there is a price jump between the two pairs that is larger than the price tolerance ε_ρ , we divide $\Delta P(i, m)$ into l segments which is the maximum integer number that is smaller than $\Delta P(i, m) / \varepsilon_g$ and calculate the marginal cost of unit at $P(i, m) + k * \varepsilon_g$ for $k = 1, \dots, l$. If the marginal cost at any new generation point is less than $\rho_g(i, m)$, the marginal cost will be substituted by $\rho_g(i, m)$ to satisfy non-decreasing conditions. Likewise, the corresponding marginal cost is substituted by $\rho_g(i, m')$ if the marginal cost of any new generation point is larger than $\rho_g(i, m')$. The newly obtained price and generation pairs could be combined with the original pairs to construct a bidding curve. For instance, assume the

marginal cost of unit in Fig. 3 is $12 + 0.1 * P$ (\$/MWh), $\varepsilon_g = 10$ MW, and $\varepsilon_\rho = \$1$ /MWh. We apply the proposed method to construct bidding curve. The generation difference between the second and third points is $100 - 60 = 40$ MW > 10 MW, so we divide the generation range between 60MW and 100MW into $l = 3$ segments. That is, the new generation points are 70, 80, 90 MW with marginal costs of 19, 20, and 21\$/MWh, respectively. The bidding curve is shown in Fig. 5.

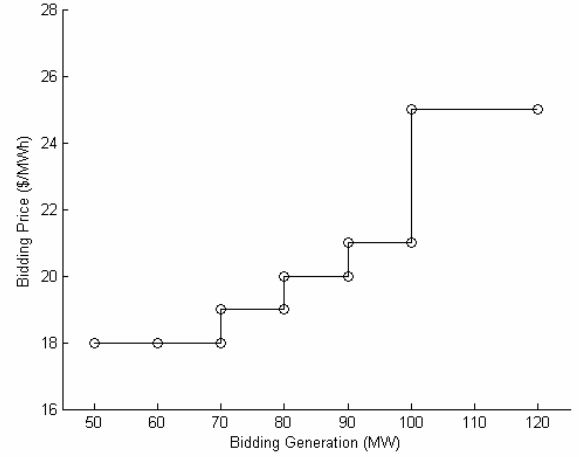


Fig.5. Method 3 to construct bidding curve

Similarly, method 4 could be developed by dividing the price difference into several segments. In the above example, we could divide the price difference between second and third points in Fig. 3 into $(25 - 18) / 1 = 7$ segments. We would obtain the new price points at 19, 20, 21, 22, 23, 24, 25 \$/MWh with the corresponding generation at 70, 80, 90, 100, 100, 100, 100 MW. Here the generation for second and third points should be no less than that for the second point and no larger than that for the third point to satisfy the non-decreasing condition. In this example, the obtained bidding curve by method 4 is the same as that shown in Fig. 5.

The above methods devise bidding curves in energy market. Similar methods for bidding in ancillary services market could be adopted based on opportunity cost.

V. RISK CONSTRAINTS

The stochastic PBUC formulation in section III is a risk-neutral model which is concerned with the optimization of expected payoff. However, a GENCO may also be concerned with its risk. Accordingly, we apply the method introduced in [25] for analyzing the risk associated with such decisions. A GENCO would set a targeted profit z_0 and the risk associated with its decision is measured by failure to meet the targeted profit. If the profit for one scenario is larger than the targeted profit, the associated downside risk would be zero; otherwise, it is the amount of unfulfilled profit. That is:

$$RISK(s) = \begin{cases} z_0 - PF(s) & \text{if } PF(s) \leq z_0 \\ 0 & \text{if } PF(s) > z_0 \end{cases} \quad (8)$$

This conditional expression is a linear constraint represented

by auxiliary binary variables as:

$$\begin{aligned} 0 &\leq RISK(s) - [z_0 - PF(s)] \leq M * [1 - x(s)] \\ 0 &\leq RISK(s) \leq M * x(s) \end{aligned} \quad (9)$$

where M is a large number. However, (9) may be deemed unnecessary based on the approach that will be introduced later. The expected downside risk for a GENCO is defined as:

$$EDR(z_0) = E[RISK(s)] = \sum_s \pi(s) * RISK(s) \quad (10)$$

The smaller the $EDR(z_0)$, the better it is for decision makers since $EDR(z_0)$ represents the profit shortfall at a targeted profit. If a decision maker is not satisfied with the risk level, a risk constraint could be added to the original formulation as:

$$EDR(z_0) \leq EDR_0 \quad (11)$$

where EDR_0 is the acceptable downside risk tolerance.

A small example is given below to show the proposed method for measuring risks. Suppose we have one thermal unit with a marginal cost of \$20/MWh and a minimum on/off time of one hour. Minimum and maximum capacity limits are 50MW and 100MW, respectively and startup and shutdown costs are ignored. The initial status of the unit is “on” for submitting bids to the energy market. The possible scenarios for hourly market price at a specific hour are shown in Table I. The optimal solution based on stochastic programming in section III would be to commit the unit with submitted generation and payoff for each scenario shown in Table II. The expected payoff in this case is:

$-250 * 0.2 - 100 * 0.2 + 200 * 0.2 + 400 * 0.2 + 500 * 0.2 = \150
If we set the targeted profit for the unit at zero, the downside risk would be:

$$EDR(0) = [0 - (-250)] * 0.2 + [0 - (-100)] * 0.2 = \$70 .$$

That is, the unit could obtain an expected payoff of \$150 with an expected downside risk of \$70. The unit which is not satisfied with the risk level will constrain the expected downside risk at a value such as $EDR(0) \leq 0 * 70 = 0$ as discussed in section III. The corresponding risk-constrained solution would be to shut down the unit with a zero payoff and zero expected downside risk in each scenario. That is, the unit could reduce its risk level at the cost of decreasing its expected payoff. Accordingly, the inclusion of risk constraint could impact the optimal solution.

The constraint on expected downside risk could result in an infeasible solution when the constraint is tight (i.e., relatively low risk tolerance EDR_0 or relatively high targeted profit). One possible approach to setting up a reasonable targeted profit and associated downside risk tolerance is to choose the profit based on operating experience or without considering risk constraint. For example, a GENCO could consider the scenario with the highest probability as its targeted profit and pick its risk tolerance. If the targeted profit is relatively high, then the downside risk tolerance should also be relatively high.

The technique introduced in [25] is to successively tighten the constraint on the expected downside risk. For instance, if the downside risk without risk constraint is $UEDR_0$, we add a

constraint such as $EDR(z_0) \leq 0.95 * UEDR_0$ and solve the resulting constrained problem with a new expected downside risk $UEDR_1$. For the next run, we tighten the risk constraint as $EDR(z_0) \leq 0.90 * UEDR_0$ and repeat this procedure until we reach an acceptable risk level. The succeeding runs in this case are viewed as independent and could be solved in parallel after solving the problem without risk constraints.

We could also penalize the expected downside risk in the objective function using a very large number. However, the advantage of the first method is that it provides a GENCO with a choice between expected payoff and risk. The second method would minimize the downside risk at the cost of reducing profit. However, the second method would only require a single execution while the first method would need multiple runs.

TABLE I
MARKET PRICE FOR THE SMALL EXAMPLE

Scenario #	Energy price (\$/MWh)	Probability
1	15	0.2
2	18	0.2
3	22	0.2
4	24	0.2
5	25	0.2

TABLE II
GENERATION AND PAYOFF FOR EACH SCENARIO

Scenario #	Generation (MW)	Payoff (\$)
1	50	-250
2	50	-100
3	100	200
4	100	400
5	100	500

In this paper, we propose the following procedure for considering risk by combining the two methods.

1. Solve the problem without considering risk constraint; Choose a target profit z_0 and calculate the expected downside risk $UEDR(z_0)$. If the calculated expected downside risk is within the GENCO's risk tolerance, stop. Otherwise, go to step (2).
2. Penalize the expected downside risk into the objective function by a very large number and solve the corresponding problem for calculating the minimum expected downside risk and the corresponding expected payoff. If the minimum expected downside risk is not acceptable, the GENCO's targeted profit is too high and a lower risk level is unattainable. The GENCO could start over with a new targeted profit. If the minimum expected downside risk is acceptable, go to step (3).
3. Choose an acceptable risk level based on the minimum expected downside risk, solve the risk-constrained problem, provide the commitment schedule of units, and devise bidding curves.

Based on the above procedure, (9) may be deemed unnecessary. In the course of implementing the above procedure, step (1) would yield a risk-neutral solution while step (2) would yield a solution with minimum risk level. The solution obtained from step (3) is a tradeoff between maximizing expected payoff and minimizing risk.

Fig. 6 shows that the bidding curve is monotonically increasing and there is a large discontinuity between 700MW and 1300MW. The discontinuity in bidding curve could incur financial losses to a GENCO as discussed before. In Fig. 7, bidding curve is refined by applying method 4 in section IV. This curve is continuous between 700MW and 1300MW which is more desirable as it will not result in significant changes in profit between any two segments.

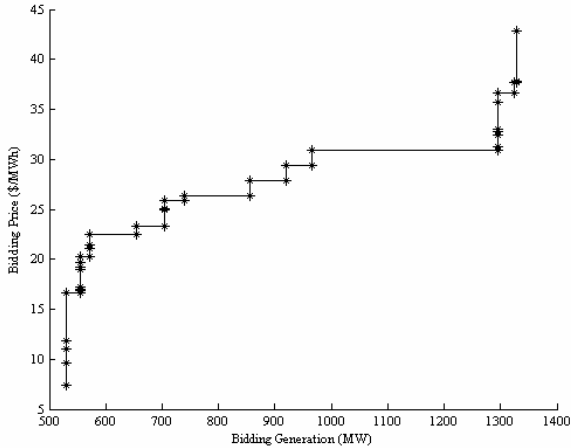


Fig. 7 Bidding curve with postprocessing at hour 8

Case 3: Impact of risk constraints

In this case, we study the impact of risk constraint on bidding curve. Table V shows the profit and probability for each scenario without risk constraint.

Table V shows that the profit for any given scenario, which is a function of hourly market price, could be quite different from those of others. For scenarios 1 and 8, the payoff is negative because the market price is quite low in these two scenarios. The GENCO would set its targeted profit at \$50,000 and those below the target are shown in bold in Table V. The corresponding probability for such scenarios is 0.2578 (i.e., $0.059 + 0.0326 + 0.0302 + 0.044 + 0.0302 + 0.0208 + 0.041$) with an expected downside risk of \$9,611.74. This risk level could be unacceptable to a GENCO. The GENCO could then apply the proposed method to control the associated risk level. Using a very large number (10,000), the expected downside risk is adjoined to the objective function for finding the minimum expected downside risk. The minimum downside risk is \$4,734.68 for the targeted profit of \$50,000 with an expected payoff of \$52,878.40. That is, for the given targeted profit, the GENCO could not expect a lower downside risk.

If a GENCO is still unsatisfied with this risk level, it means that the GENCO's targeted profit is too high. Accordingly, we assume the acceptable expected downside risk level for the GENCO is \$6,000. By considering risk-constrained model, the objective would be \$59,591.40 with a downside risk of \$6,000. Accordingly, a GENCO could reduce its downside risk level by 37.59% ($(9,611.74 - 6,000) / 9,611.74$) at the cost of reducing its expected payoff by 4.31% ($(62,278.50 - 59,591.40) / 62,278.50$). Table VI shows the payoff for each scenario with risk constraint in which scenarios with increased profit are shown in bold. The portion of unit schedule that is different from that in Table IV is shown in Table VII.

The comparison of Tables V and VI shows that the profit for scenarios 1 and 8 is no longer negative when units 1005, 1010, and 1011 are shut down at additional hours. The additional payoff for scenarios in Table VI is at the cost of reducing payoffs for other scenarios. The comparison of Tables IV and VII shows that units 1005, 1010, and 1011 are shut down at additional hours when the commitment of these units would contribute to the maximization of expected payoff but would also increase the downside risk. The GENCO could shut down the units for reducing its expected downside risk at the cost of reducing the expected payoff.

It should be noted that the inclusion of risk constraint would impact hourly bidding curve. For the purpose of comparison, we only show in Fig. 8 the bidding curve at hour 8. Comparison of Figs. 7 and 8 shows that the risk constraint would reduce the offered generation based on bidding price. The major reduction is because the relatively large units 1010 and 1011 are shut down at hour 8. However, the dispatch of smaller units at hour 8 also contributes to the reduction of total generation offered by the GENCO.

TABLE V
SCENARIO PROFIT WITHOUT RISK CONSTRAINTS

#	Profit (\$)	Probability	#	Profit (\$)	Probability
1	-13936.20	0.0590	16	118743	0.0352
2	87408.80	0.0262	17	12982.60	0.0440
3	59681.60	0.0418	18	121046	0.0382
4	70304.50	0.0382	19	86731.40	0.0266
5	48108.40	0.0326	20	53894.10	0.0282
6	121253	0.0410	21	61158.30	0.0374
7	61557.40	0.0278	22	63617.20	0.0302
8	-7847.60	0.0302	23	44169.20	0.0302
9	96364.10	0.0278	24	37966.70	0.0208
10	78337.80	0.0216	25	77150	0.0224
11	12164.20	0.0260	26	57976.60	0.0318
12	1227810	0.0384	27	35200.40	0.0410
13	38994.90	0.0350	28	137145	0.0230
14	59412.10	0.0312	29	71658.60	0.0302
15	66565.70	0.0410	30	55793.20	0.0430

TABLE VI
SCENARIO PROFIT WITH RISK CONSTRAINTS

#	Profit (\$)	Probability	#	Profit (\$)	Probability
1	8126.85	0.0590	16	95971.70	0.0352
2	72822.30	0.0262	17	22815.50	0.0440
3	51017.30	0.0418	18	100178	0.0382
4	61206.40	0.0382	19	72121.50	0.0266
5	48815	0.0326	20	49087.50	0.0282
6	105478	0.0410	21	49218.20	0.0374
7	52556	0.0278	22	57947.90	0.0302
8	5835.51	0.0302	23	52398.20	0.0302
9	87282.90	0.0278	24	50381.60	0.0208
10	65686	0.0216	25	69806.40	0.0224
11	24759	0.0260	26	58741.50	0.0318
12	105690	0.0384	27	43909.10	0.0410
13	50984.80	0.0350	28	120670	0.0230
14	51632.80	0.0312	29	71458.90	0.0302
15	69763.90	0.0410	30	57646	0.0430

TABLE VII
UNIT SCHEDULE WITH RISK CONSTRAINTS

Unit	Hours (0-24)																										
1005	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
1010	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1011	1	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0

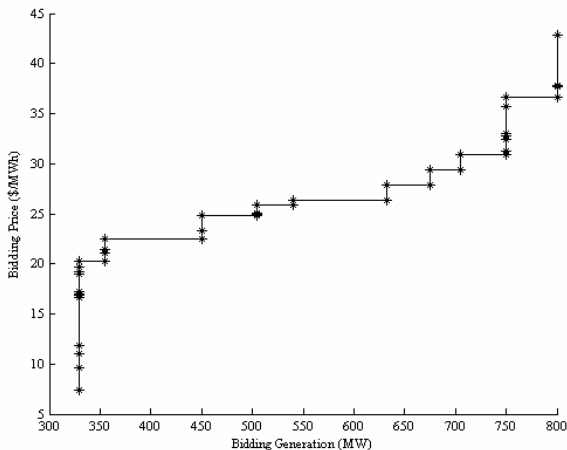


Fig. 8 Bidding curve for hour 8 with risk constraint

VII. DISCUSSION

It should be noted that there may be coupling among consecutive hourly prices. For example, if the market price at one hour is high, it is probable that the next hour price would also be high. In this paper, we run the Monte Carlo simulation for individual hours and plan to consider the coupling among hourly market prices in our future studies. However, different market price simulation approaches would only impact the input to our proposed formulation.

The proposed model could be applied to a large system as our formulation depends on commercial MIP solvers for solving the stochastic mixed integer program. With the further developments in both hardware and software algorithms, leading commercial MIP solvers such as CPLEX, OSL, XPRESS, and LINDO have been improved significantly for solving very large cases [11]. Stochastic Lagrangian relaxation (LR) [26], [27] could also be applied for solving the proposed formulation. However, the MIP formulation has the following advantages over the LR approach [11]: (1) global optimality; (2) direct measure of the optimality of a solution; (3) more flexible and accurate modeling capabilities. We chose to apply the commercial MIP solver to solve the GENCO's problem since the size of the problem is generally within the solving capability of commercial solvers.

The input data to our formulation include market prices for energy and ancillary services and unit technical data such as cost curve, minimum on/off times, ramping up/down limits, etc. Market prices and variances could be forecasted by available forecasting techniques such as artificial neural network, time series model [1]. The maintenance of the input data is easy and flexible.

The proposed formulation is very practical which could be applied by GENCOs for submitting offers to energy and ancillary services markets. Meanwhile, the possible inclusion of arbitrage strategies [1], [27], [28] in the proposed model would provide a very practical GENCO tool for maximizing portfolios in energy, bilateral contracts, ancillary services, fuel, and emission allowance markets.

VIII. CONCLUSIONS

A risk-constrained bidding strategy for day-ahead energy and ancillary services markets is proposed for GENCOs. The market price uncertainty is modeled using scenarios and scenario reduction techniques are applied to reduce the number of scenarios in consideration. The risk associated with the market price uncertainty is modeled using the downside target profit shortfall and is incorporated explicitly as a constraint in the model. Accordingly, a closed-loop bidding strategy is constructed. After solving the stochastic PBUC problem, postprocessing is applied based on marginal costs for refining bidding curves.

Test results illustrate that it is necessary to consider market price uncertainty and incorporate the impact of stochastic market price on the commitment schedule of units. It is also shown that risk constraints would play an important role in devising bidding curves. A GENCO could significantly reduce its risk level at the cost of reducing expected payoffs.

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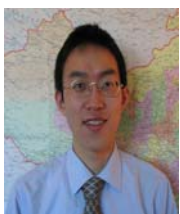
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