Reliability Analyses of Wide-Area Protection Systems Considering Cyber-Physical System Constraints

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Appendix

This appendix presents the derivations of (8), (10), and (13) using Permutation and Combination.

A. Probability of Fault Initiation

Here, $S^0 = \delta card(\mathrm{U}^0_{(1)})$ is the bus sequence voltage data uploaded by substations; δ is the data redundancy; $S = \lfloor S^0 d \rfloor$ is the data flow received by the control center at a given *d*, and Υ is the total number of buses which meet the initiation setting value of sequence voltage when a fault occurs.

If $S = 0$, it is obvious that no fault related information can be obtained; so the probability of accurately initiating WAPS is 0. And if $0 < S < S^0 - \Upsilon\delta + 1$, the WAPS process will be initiated when any bus sequence voltage is less than the preset initiation setting value. When a bus sequence voltage which satisfies the conditions is taken out, the number of events containing this bus sequence voltage is $C^{S-1}_{S^0-1}$; when the next bus sequence voltage which satisfies the conditions is taken out, the number of events containing this bus sequence voltage and excluding the bus sequence voltage previously taken is $C^{S-1}_{S^0-2}$, and so on. We continue this process until all bus sequence voltages which satisfy the conditions are taken out, where the sum shows the number of events in which at least one bus sequence voltage is less than the initiation setting value in terms of $\sum_{i=1}^{\Upsilon\delta} C^{S-1}_{S^0-i}$. The number of elementary events receiving S bus sequence voltages is $C^{S}_{S^0}$. Therefore, the probability of accurately initiating WAPS is $\sum_{i=1}^{\Upsilon\delta} C^{S-1}_{S^0-i} \Big/ C^{S}_{S^0}$. Then $S^0 - \Upsilon\delta + 1 \le S \le S^0$, where it is apparent that at least one bus sequence voltage satisfying the conditions must be received; so the probability of accurately initiating WAPS is 1.

B. Probability of Fault Area Positioning

Moreover, $\alpha \in \{1,2\}$ represents the number of bus sequence voltage on the sending or the receiving end of a transmission line which satisfies the initiation criteria. If $S = 0$, it is obvious that no fault related information can be obtained; so the probability of accurately positioning the fault area is 0. And if $0 < S < S^0 - \alpha\delta + 1$, the fault area will be positioned when any bus sequence voltage of two ends is less than the setting value. When a bus sequence voltage satisfying the conditions is taken out, the number of events containing this bus sequence voltage is $C^{S-1}_{S^0-1}$; when the next bus sequence voltage satisfying the conditions is taken out, the number of events containing this bus sequence voltage and excluding the bus sequence voltage previously taken is $C^{S-1}_{S^0-2}$, and so on. We continue this process until all the bus sequence voltages meeting the conditions are taken out, where the sum shows the number of events that at least one bus sequence voltage is less than the initiation setting value in terms of $\sum_{i=1}^{\alpha\delta} C^{S-1}_{S^0-i}$. The number of elementary events accurately initiating WAPS is $\sum_{i=1}^{\Upsilon\delta} C^{S-1}_{S^0-i}$. Therefore, the probability of accurately positioning the fault area is $\sum_{i=1}^{\alpha\delta} C^{S-1}_{S^0-i} \Big/ \sum_{i=1}^{\Upsilon\delta} C^{S-1}_{S^0-i}$. Then $S^0 - \alpha\delta + 1 \le S \le S^0$, where it is apparent that at least one bus sequence voltage of two ends satisfying the conditions must be received; so the probability of accurately positioning the fault area is 1.

C. Probability of Faulted Line Recognition

Here, $Q^0 = 2\delta card(\mathrm{L})$ is the total amount of uploaded data of directional elements in area Λ; δ is the data redundancy; $Q = \lfloor Q^0 d \rfloor$ is the amount of the direction data obtained by the control center; and $\beta := Q - (Q_0 - 2\delta)$. If $Q < 2$, it is obvious that at least one end must have the direction data lost, so the probability of accurately recognizing the faulted line is 0.

If $2 \le Q < \delta$, the number of events without the information on faulted line or with one specific end receiving the direction data is $\sum_{i=0}^{Q} C^{i}_{\delta} C^{Q-i}_{Q^0-2\delta}$, where C^{i}_{δ} is to take *i* data from the information at specific end of the faulted line, and $C^{Q-i}_{Q^0-2\delta}$ is to take *Q-i* data from the information on non-faulted line. Thus, the number of events without the information on faulted line or with either one end receiving the direction data is $2\sum_{i=0}^{Q} C^{i}_{\delta} C^{Q-i}_{Q^0-2\delta} - C^{Q}_{Q^0-2\delta}$, where $C^{Q}_{Q^0-2\delta}$ is the number of events without faulted line information. Because the repetition is accumulated, it needs to be subtracted from the sum. And the number of elementary events uploaded in area Λ and receiving Q direction data is $C^{Q}_{Q^0}$. Therefore, the probability of accurately recognizing the faulted line is

$1-2\sum_{i=0}^{Q} C_{\delta}^{i} C_{Q^0-2\delta}^{Q-i} - C_{Q^0-2\delta}^{Q} \Big/ C_{Q^0}^{Q}$.

If $\delta \le Q \le Q^0 - 2\delta$, as in previous analysis, it is concluded that the probability of accurately recognizing the faulted line is $1-2\sum_{i=0}^{\delta} C_{\delta}^{i} C_{Q^0-2\delta}^{Q-i} - C_{Q^0-2\delta}^{Q} \Big/ C_{Q^0}^{Q}$. If $Q^0 - 2\delta < Q \le Q^0 - \delta$, the number of events that can receive the direction data of one end but not both ends is $2\sum_{i=\beta}^{\delta} C_{\delta}^{i} C_{Q^0-2\delta}^{Q-i}$. And the number of elementary events uploaded in area Λ and receiving Q direction data also is $C_{Q^0}^{Q}$. Therefore, the probability of accurately recognizing the faulted line is $1-2\sum_{i=\beta}^{\delta} C_{\delta}^{i} C_{Q^0-2\delta}^{Q-i} \Big/ C_{Q^0}^{Q}$. Then $Q^0 - \delta < Q \le Q^0$; it is apparent that the direction angle data at both ends of the faulted line must be received; so the probability of accurately recognizing the faulted line is 1.