APPENDIX

A. *Final formulation of the Planning Problem*

After linearization, the objective of the planning problem is stated in (1), which is subject to investment (2)-(4) and operation (5)-(43) constraints.

Objective $$\min\left(TOC+TIC\right) \tag{1}$$

S.t.

$$TIC = f_N \times \mathbf{IC} \tag{2}$$

$$TOC = \mathbf{MC}\times f_N + \sum_{s=1}^{S}\rho(s)\sum_{\tau}^{\Gamma}C_{bio}m_{bio}(\tau,s) \tag{3}$$

$$0\le f_N \le f_N^{\max} \tag{4}$$

$$Rel_j = \frac{\sum_{s=1}^{S}\rho(s)\sum_{\tau=1}^{24}z_j(\tau,s)\left(L_j^{'}(\tau,s)-L_j(\tau,s)\right)}{L_j^{'}(\tau,s)} \le \lambda_j \tag{5}$$

$$L_e(\tau,s)=E_{CHP}(\tau,s)+E_{PV}(\tau,s)-E_{BSS}^{-}(\tau,s)+E_{BSS}^{+}(\tau,s) \tag{6}$$

$$L_{th}(\tau,s)=H_{TES}^{-}(\tau,s) \tag{7}$$

$$L_g(\tau,s)=Q_{CH_4}\left(G_{BES}^{-}(\tau,s)-G_{CHP}(\tau,s)\right) \tag{8}$$

$$T_d(\tau+1,s)=T_d(\tau,s)+\frac{H_{heat}(\tau,s)-H_m(\tau,s)-H_{sur}(\tau,s)}{c_m\rho_m V_{AD}} \tag{9}$$

$$H_{heat}(\tau,s)=\eta_{loss}\left(H_{AD}^{CHP}(\tau,s)+H_{AD}^{SWH}(\tau,s)\right) \tag{10}$$

$$H_m(\tau,s)=c_m m_{bio}(\tau,s)\left(T_{air}(\tau,s)-T_d(\tau,s)\right) \tag{11}$$

$$H_{sur}(\tau,s)=\sum_{i=1}^{I}N_i A_i\left(T_d(\tau,s)-T_s(\tau,s)\right) \tag{12}$$

$$m_{bio}(\tau,s)\ge\eta_{bio}G_{bio}(\tau,s) \tag{13}$$

$$E_{PV}(\tau,s)=f_{PV}N_{PV}P_0\frac{J(\tau,s)}{J_{STC}}\left(1+\partial_T\left(T_{air}(\tau,s)-T_{STC}\right)\right)\Delta\tau \tag{14}$$

$$H_{SWH}(\tau,s)=\frac{1}{f_{SWH}}N_{SWH}\eta_{SWH}J(\tau,s)(1-\eta_{loss}) \tag{15}$$

$$H_{SWH}(\tau,s)=H_{TES}^{SWH}(\tau,s)+H_{AD}^{SWH}(\tau,s) \tag{16}$$

$$\eta_{CHP}^{\min}Q_{CH_4}G_{CHP}(\tau,s)\le E_{CHP}(\tau,s)+H_{CHP}(\tau,s)\le\eta_{CHP}^{\max}Q_{CH_4}G_{CHP}(\tau,s) \tag{17}$$

$$E_{CHP}(\tau,s)+H_{CHP}(\tau,s)\le N_{CHP} \tag{18}$$

$$H_{CHP}(\tau,s)=H_{AD}^{CHP}(\tau,s)+H_{TES}^{CHP}(\tau,s) \tag{19}$$

$$E_{BSS}(\tau+1,s)=E_{BSS}(\tau,s)+\eta_{BSS}^{+}E_{BSS}^{+}(\tau,s)-\frac{E_{BSS}^{-}(\tau,s)}{\eta_{BSS}^{-}} \tag{20}$$

$$0\le E_{BSS}(t,s)\le N_{BSS} \tag{21}$$

$$E_{BSS}^{+}(\tau,s)\le\beta_{BSS}^{\max}N_{BSS} \tag{22}$$

$$E_{BSS}^{-}(\tau,s)\le\beta_{BSS}^{\max}N_{BSS} \tag{23}$$

$$E_{BSS}(t=1,s)\le E_{BSS}(t=24,s) \tag{24}$$

$$G_{BES}^{+}(\tau,s)=G_{bio}(\tau,s) \tag{25}$$

$$G_{BES}(\tau+1,s)=G_{BES}(\tau,s)+\eta_{BES}^{+}G_{BES}^{+}(\tau,s)-\frac{G_{BES}^{-}(\tau,s)}{\eta_{BES}^{-}} \tag{26}$$

$$\alpha_{BES}^{\min}N_{BES}\le Q_{CH_4}G_{BES}(\tau,s)\le\alpha_{BES}^{\max}N_{BES} \tag{27}$$

$$G_{BES}(t=1,s)\le G_{BES}(t=24,s) \tag{28}$$

$$Q_{CH_4}G_{BES}^{+}(\tau,s)\le\beta_{BES}^{\max}N_{BES} \tag{29}$$

$$Q_{CH_4}G_{BES}^{-}(\tau,s)\le\beta_{BES}^{\max}N_{BES} \tag{30}$$

$$H_{TES}(\tau+1,s)=H_{TES}(\tau,s)+\eta_{TES}^{+}H_{TES}^{+}(\tau,s)-\frac{H_{TES}^{-}(\tau,s)}{\eta_{TES}^{-}} \tag{31}$$

$$\alpha_{TES}^{\min}N_{TES}\le H_{TES}(\tau,s)\le\alpha_{TES}^{\max}N_{TES} \tag{32}$$

$$H_{TES}^{+}(\tau,s)\le\beta_{TES}^{\max}N_{TES} \tag{33}$$

$$H_{TES}^{-}(\tau,s)\le\beta_{TES}^{\max}N_{TES} \tag{34}$$

$$G_{bio}(T_d)=y_1G_{bio}(b_1)+y_2G_{bio}(b_2)+\ldots+y_mG_{bio}(b_m) \tag{35}$$

$$y_1\le z_1, y_2\le z_1+z_2,\ldots,y_{m-1}\le y_{m-2}+y_{m-1}, y_m\le y_{m-1} \tag{36}$$

$$y_1+y_2+\ldots+y_m=1\quad\forall\tau,s \tag{37}$$

$$z_1+z_2+\ldots+z_{m-1}=1,\ \forall\tau,s \tag{38}$$

$$T_d=y_1b_1+y_2b_2+..,+y_mb_m,\ \forall\tau,s \tag{39}$$

$$L_j^{'}-L_j\le M\left(1-z_j\right),\quad\forall\tau,s,j \tag{40}$$

$$L_j^{'}-L_j\le z_jM,\quad\forall\tau,s,j \tag{41}$$

$$z_j\Phi\le r_j\le z_jM,\quad\forall\tau,s,j \tag{42}$$

$$0\le L_j-r_j\le\left(1-z_j\right)M,\ \forall\tau,s,j \tag{43}$$

B. *Application of Benders Decomposition*

The proposed planning problem is restated as

$$\begin{aligned}&\min \boldsymbol{C}^T\boldsymbol{x}+\sum_{s\in\boldsymbol{S}}\boldsymbol{P}(s)\boldsymbol{D}^T\boldsymbol{y}_s\\ &s.t.\ \ \boldsymbol{Ex}+\boldsymbol{Fy}_s+\boldsymbol{Gz}_s\ge\boldsymbol{h};\\ &\quad\ \boldsymbol{Ax}\ge\boldsymbol{b};\boldsymbol{x}\ge0,\boldsymbol{y}_s\ge0,\boldsymbol{z}_s\in(0,1);\end{aligned} \tag{44}$$

where $\boldsymbol{x}$ denotes $\boldsymbol{f_N}$ at the investment stage, $\boldsymbol{y_s}$ represents continuous variables in the operation stage; $\boldsymbol{z_s}$ includes binary variables introduced by the linearization in the operation stage; $P(s)$ is the expectation, and s denotes a scenario; $\boldsymbol{Ax}\ge\boldsymbol{b}$ refers to the investment constraint (2)-(4); $\boldsymbol{Ex}+\boldsymbol{Fy_s}+\boldsymbol{Gz_s}\ge\boldsymbol{h}$ collects all operation constraints (5)-(43).

Although this planning problem can be solved as one large-scale linear programing problem, the computation burden will be significant when a large number of scenarios are involved. Here, we adopt the Benders decomposition to decompose the original large-scale planning problem into a master investment problem and two sets of operation subproblems.

1) *Master problem*

The formulation of the master problem with Benders cuts which are generated iteratively from the subproblem is stated in (45). At each iteration n, we solve the master problem to obtain the lower bound LB_n, which is equal to the optimal objective value of (45).

$$\begin{aligned}&\min \boldsymbol{C}^T\boldsymbol{x}+\boldsymbol{\theta}\\ &\boldsymbol{s.t.}\ \ \boldsymbol{Ax}\ge\boldsymbol{b},\boldsymbol{x}\ge0,\boldsymbol{\theta}\ge0\end{aligned} \tag{45}$$

2) *Feasible subproblem*

The feasible subproblem will check whether the investment decision obtained in the master problem is feasible when applied to the operation stage. For each scenario, given the optimal $\boldsymbol{x}^*$ value of the master problem, the feasible subproblem is stated in (46). Note that this subproblem is non-convex that cannot directly provide the Benders cut for the master problem (45). The modified feasible subproblem is reformulated as in (47) after solving (46) to obtain the binary $\boldsymbol{z}_s^*$ and fixing its value in (51).

$$\begin{gathered}\min \boldsymbol{1}^{\Gamma} \boldsymbol{s} \\ s.t.\ \boldsymbol{F}\boldsymbol{y}_s + \boldsymbol{s} \geq \boldsymbol{h} - \boldsymbol{E}\boldsymbol{x}^* - \boldsymbol{G}\boldsymbol{z}_s, \boldsymbol{y}_s \geq 0\end{gathered} \tag{46}$$

$$\begin{gathered}\min \boldsymbol{1}^{\Gamma} \boldsymbol{s} \\ s.t.\ \boldsymbol{F}\boldsymbol{y}_s + \boldsymbol{s} \geq \boldsymbol{h} - \boldsymbol{E}\boldsymbol{x}^* - \boldsymbol{G}\boldsymbol{z}_s^*, \boldsymbol{y}_s \geq 0\end{gathered} \tag{47}$$

If the objective value of (47) is not zero, a feasibility cut (48) is generated and added to the master problem, where $\boldsymbol{v}_s^*$ is the optimal dual solution to (47).

$$\boldsymbol{0} \geq \sum_{s\in S} p(s)\left(\boldsymbol{h} - \boldsymbol{E}\boldsymbol{x} - \boldsymbol{G}\boldsymbol{z}_s^*\right)^{\Gamma} \boldsymbol{v}_s^* \tag{48}$$

3) Optimal subproblem

If the objective value of (47) is equal to zero, the optimal subproblem is stated as (49).

$$\begin{gathered}\min \boldsymbol{D}^{T} \boldsymbol{y}_s \\ s.t.\ \boldsymbol{E}\boldsymbol{x}^* + \boldsymbol{F}\boldsymbol{y}_s + \boldsymbol{G}\boldsymbol{z}_s \geq \boldsymbol{h}, \boldsymbol{y}_s \geq 0\end{gathered} \tag{49}$$

The same method used in solving the feasible problem is adopted to solve the optimal problem. Given the optimal primal solution $\boldsymbol{y}_s^*$ and the optimal dual solution $\boldsymbol{u}_s^*$ of (50), the upper bound is updated as (50).

$$UB_{n+1} = \min\left\{UB_n,\ \boldsymbol{C}^T \boldsymbol{x} + \sum_{s\in S} \boldsymbol{P}(s)\boldsymbol{D}^T \boldsymbol{y}_s^*\right\} \tag{50}$$

If $|UB_n\text{-}LB_n| < \varepsilon$ cannot be satisfied, the optimal cut will be generated and added to (45) at the next iteration; otherwise the process will be terminated.

$$\boldsymbol{\theta} \geq \sum_{s\in S} p(s)\left(\boldsymbol{h} - \boldsymbol{E}\boldsymbol{x} - \boldsymbol{G}\boldsymbol{z}_s^*\right)^{\Gamma} \boldsymbol{u}_s^*, \tag{51}$$

C. Proposed Solution Procedure

Using the stated strategy for the planning problem, the solution stapes are itemized as follows.

Step 1: Initialize all parameter values

Step 2: Solve the initial master problem $\min \boldsymbol{C}^T \boldsymbol{x}$, s.t. $\boldsymbol{A}\boldsymbol{x} \geq b, \boldsymbol{x} \geq 0$. Obtain the optimal value $\boldsymbol{x}^*$ and state the initial solution $\boldsymbol{\mu}^*$. LB=$\boldsymbol{\mu}^*$.

Step 3: For each scenario, solve the feasible subproblem.

3.1 Solve (45) to obtain $\boldsymbol{z}_s^*$, then solve (46) to obtain $\boldsymbol{v}_s^*$. If the objective of (47) is equal to zero, go to step 4. Otherwise, go to step 3.2

3.2 Generate the feasibility cut (48), add it to the master problem (45), and go to step 4.

Step 4: for each scenario, solve the optimal subproblem.

4.1 Solve (49) to obtain $\boldsymbol{z}_s^*$, $\boldsymbol{y}_s^*$ and $\boldsymbol{u}_s^*$.

4.2 Update the upper bound using (50). If $|UB_n\text{-}LB_n| < \varepsilon$, stop the iterations; otherwise go to step 4.3.

4.3 Generate the optimality cut (51) to (45). Go to step 5 and repeat the process.

Step 5 Solve the master problem (45) to obtain the updated $\boldsymbol{x}^*$ and LB, then repeat step 3.